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Magnetic quantum oscillations of diagonal conductivity in a two-dimensional conductor with a weak square superlattice modulation under conditions of the integer quantum Hall effect

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Abstract. We report on analytical and numerical studies of the magnetic quantum oscillations of the diagonal conductivity σ_{xx} in a two-dimensional conductor with a weak square superlattice modulation under conditions of the integer quantum Hall (IQHE) effect. The quantum Hall effect in such a system differs from the conventional IQHE, in which the finite width of the Landau bands is due to disorder only. The superlattice modulation potential yields a fractal splitting of the Landau levels into Hofstadter minibands. For rational flux through a unit cell, the minibands have a finite width and intrinsic dispersion relations. We consider a regime, now accessible experimentally, in which disorder does not wash out the fractal internal gap structure of the Landau bands completely. We found the following distinctions from the conventional IQHE produced by the superlattice: (i) the peaks in diagonal conductivity are split due to the Hofstadter miniband structure of Landau bands; (ii) the number of split peaks in the bunch, their positions and heights depend irregularly on the magnetic field and the Fermi energy; (iii) the gaps between the split Landau bands (and related quantum Hall plateaus) become narrower with the superlattice modulation than without it.

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Contents

1. Introduction

According to the theoretical predictions [1, 2], done soon after the discovery of the integer quantum Hall effect (IQHE) [3], the Hall conductivity σ_{xy} in a two-dimensional (2D) conductor with a weak superlattice modulation increases non-monotonically because of the complex fractal internal structure of the Landau bands known in the literature as the Hofstadter butterfly [4–7]. In other words, a non-monotonic sequence of quantum Hall plateaus with increasing magnetic field was predicted. This means that the Hall conductance varies in a nontrivial way from minigap to minigap in the Hofstadter fractal band structure.

The Hall conductivity at zero temperature, as was shown in [2], can be expressed as the sum of two terms: $\sigma_{xy} = \sigma_{xy}^{\mathrm{C}} + \sigma_{xy}^{\mathrm{Q}}$. In a model of free electrons the first term corresponds to the classical value of the Hall conductivity $\sigma_{xy}^{\mathrm{C}} = \omega_{\mathrm{c}} \tau \sigma_{xx} \propto B$ ($\omega_{\mathrm{c}} = eB/m_{\mathrm{e}}c$ is the cyclotron frequency and τ is the electron scattering time). The second term σ_{xy}^{Q} is purely quantum and has no classical interpretation. Within the minigaps of the Hofstadter spectrum, $\sigma_{xy}^{\mathrm{C}} = 0$, so that $\sigma_{xy} = \sigma_{xy}^{\mathrm{Q}}$. The diagonal conductivity also vanishes within the minigaps, $\sigma_{xx} = 0$, while the quantum Hall conductance is quantized according to the rule $\sigma_{xy}^{\mathrm{Q}} = 2e^2(\sigma + n)/h$. Here the factor two is due to the spin, n is the Landau band index and the index

$$\sigma = \frac{hc}{e} \frac{\partial N}{\partial B} \qquad (1)$$

can be calculated from the integrated density of states $N(E, B)$ within the gap. This index takes positive or negative integer values depending on the minigap in question [1, 2]. Such behavior of the σ_{xy} differs from the monotonic step-like increase of the Hall conductivity with increasing magnetic field typical for the conventional (without the superlattice) IQHE [8, 9], which corresponds to the limit $\sigma = 0$. It is worth noting here that the σ in equation (1) and related $\sigma_{xy}^{\mathrm{Q}} = 2e^2(\sigma + n)/h$ are thermodynamic quantities in the sense that they are completely determined by the energy spectrum through the integrated density of states $N(E, B)$. On the contrary, the diagonal conductivity σ_{xx} is sensitive not only to the energy spectrum but also to the mechanisms of electron propagation through a disorder potential, which are closely related to the complex problem of localization in 2D systems. Different views and various nontrivial thermodynamic effects in the 2D periodically modulated electron system in perpendicular quantizing magnetic field have been discussed in the literature (see [10]–[13] and references therein).

A viewpoint on the Hofstadter butterfly as a quantum phase diagram with infinite phases labeled by their quantized Hall conductances was developed in [10]. The fractal properties of the magnetization oscillations as a function of the chemical potential in the Hofstadter 2D systems without disorder and at zero temperature have been studied in [11]. The lip-shaped envelope of these magnetization oscillations on all scales was also established in this paper. The Hofstadter spectrum effects in de Haas–van Alphen (dHvA) oscillations of a 2D square lattice electron system have been studied in [12, 13]. These thermodynamic quantum oscillations display rather nontrivial features in the Fourier spectrum caused by the magnetic-breakdown-mixing of the electron and hole orbits. The disorder acts on the dHvA harmonics through the standard Dingle factors, which suppress the thermodynamic oscillations.

The influence of a weak disorder on the diagonal conductivity in the presence of a 2D superlattice modulation is more complex because of the localization of electrons and interference of their transport with the Hofstadter spectrum effects. The electron localization is absent in perfect periodic 2D superlattices. In contrast, in 2D spatial fractal structures like the Sierpinski gasket, electrons are localized even without any disorder or structural imperfections [14]. It was shown in [15] that a magnetic field applied perpendicular to these fractals breaks the lattice symmetry of the Sierpinski gasket and delocalizes electrons. In regular 2D superlattices, the role of the magnetic field due to the rearrangement of the Landau bands into Hofstadter butterflies is very important too. The corresponding fractal effects in oscillations of the diagonal and Hall conductivities have been found experimentally in 2D GaAs/AlGaAs heterostructures with a weak 2D lateral potential modulation [16]–[18]. This became possible due to the advances in 2D superlattice fabrication.

Advances in a theoretical description of these oscillations are related only to different thermodynamic aspects of the problem. A theoretical description of the diagonal conductivity oscillations under conditions of the IQHE with Landau bands having a fractal internal structure of the Hofstadter butterfly has not been published so far to the best of our knowledge. The present paper is aimed toward filling this gap.

Magnetic field plays a crucial role in the diagonal conductivity σ_{xx} of noninteracting 2D electrons. It is believed that without magnetic field all states are localized in 2D disordered systems at zero temperature [19]. In the presence of quantizing magnetic field the energy spectrum consists of the disorder-broadened Landau bands with narrow stripes of extended states in the middle, which are responsible for the nonzero diagonal conductivity σ_{xx} in the IQHE [8, 9]. The conductivity σ_{xx} has peaks in transitional plateau-to-plateau regions where the Hall conductivity σ_{xy} jumps to an adjacent plateau. Both conductivities, σ_{xx} and σ_{xy}, demonstrate a scaling behavior in transitional regions in clean samples, but this scaling is absent in dirty samples. A possible explanation for the phenomenon was given in a recent paper [20] within the modified two-channel network model in which peaks and valleys of the smooth disorder potential comprise a 2D periodic structure [21, 22]. It was shown numerically in [20] that the width of extended states goes to zero when the overlap of Landau bands is below a critical value. Above this critical value (i.e. with increasing disorder) extended states at the center of Landau bands evolve gradually into a stripe of finite width in the middle between the adjacent overlapping Landau bands. A similar effect was found in numerical simulations of the paper [23] in which the Hall plateau diagram in the lowest Landau level (LL) split by a 2D periodic potential into the Hofstadter butterfly has been studied on the basis of equation (1) in a model with random $\delta-$ potential. It was shown that randomly distributed point impurities smear the plateau-to-plateau transitions making flat plateaus between them shorter with the increase of

disorder potential. The localization length, calculated from the size dependence of the Thouless number [24], was found to be infinite only for energies in the middle of the plateau-to-plateau transitions, while the rest of the states are localized. With increasing disorder the Hofstadter minibands smear and merge into bunches. Correspondingly, the extended states of the minibands evolve toward the center of the bunches, while minigaps between them and related plateaus in the σ_{xy} vanish gradually (for details and more references on the localization in the Hofstadter spectrum see [23]).

The above picture is in qualitative agreement with the experimental observations of the paper [18] in which it was found that peaks in the diagonal conductivity σ_{xx} strongly correlate with the plateau-to-plateau transitions of the Hall conductivity σ_{xy}. In conventional IQHE such a type of correlation between the Hall and diagonal conductivities is well established and regulated by the semicircle rule [25]. It is worth noting here that the quantity σ_{xy}^{Q}, calculated numerically in [23] is only a thermodynamic part of the total Hall conductivity $\sigma_{xy} = \sigma_{xy}^{C} + \sigma_{xy}^{Q}$. The term σ_{xy}^{C} depends not only on the energy spectrum, but also on σ_{xx}, which is sensitive to disorder and localization, and vanishes wherever $\sigma_{xx} = 0$ as one can see from the quasiclassical limit $\sigma_{xy}^{C} = \omega_c \tau \sigma_{xx}$ [2].

In this paper, we calculate σ_{xx} as a function of the magnetic field B and compare results with the experiment of the paper [18] in which the resistivity $\rho_{xx}(B) = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{xy}^2)$ and the Hall conductivity have been measured concurrently. The Hall plateau boundaries can be found as well from the diagonal conductivity, because σ_{xx} equals zero within the plateaus and has peaks in transitional regions between the plateaus.

We consider a 2D conductor with a weak square superlattice modulation under conditions of the IQHE assuming that a disorder is small enough to suppress the fractal fine structure of the Landau bands completely. The states within each miniband are expected to be localized, except for a thin stripe of delocalized states at the expectation value of the energy of each band. The number of localized states is assumed to be proportional to the total number of states in the miniband, and the Fermi energy independent of the magnetic field. We found that the effect of the superlattice in the IQHE regime results in a nontrivial splitting of the peaks in the $\sigma_{xx}(B)$ into bunches of peaks. The bunches are separated by plateau regions where $\sigma_{xx}(B) = 0$. The number of peaks in a bunch and distances between them depend on the Hofstadter miniband structure, which is governed by the position of the Fermi energy and the value of the magnetic field. All these features are in good qualitative agreement with the experimental observations of the paper [18].

2. Basic equations

2.1. Fractal Landau bands caused by a weak square superlattice modulation of a 2D electron gas

The energy spectrum of 2D electrons subject to a weak potential of the form

$$V(x, y) = \frac{V_0}{4} \left[\cos(2\pi x/a) + \cos(2\pi y/a)\right] \tag{2}$$

and a magnetic field can be written as [6, 18, 26]

$$E_{n,m} = \hbar\omega_c(n + (1/2)) + \frac{V_0}{8} \widetilde{E}_m \left(\frac{\Phi_0}{\Phi}\right) \exp\left(-\frac{\pi}{2}\frac{\Phi_0}{\Phi}\right) \left|L_n\left(\pi\frac{\Phi_0}{\Phi}\right)\right|, \tag{3}$$

where $n = 0, 1, 2, \ldots$ is the LL index and $\tilde{E}_m$ stands for the energies of the mth miniband of the Hofstadter butterfly. The first term here is the Landau energy spectrum with the cyclotron frequency $\omega_c = h\Phi/mA_c\Phi_0$, where A_c is the area of the unit cell. The second term in (3) depends on the inverse number of flux quanta per unit cell Φ_0/Φ, where $\Phi_0 = hc/e$ is the flux quantum. The Hofstadter butterfly is a periodic function in the flux quantum number $\tilde{E}_m(x) = \tilde{E}_m(x+1)$. The width of the Landau bands is modulated by the Laguerre polynomials $L_n(\pi\Phi_0/\Phi)$, which are oscillating functions in the ratio Φ_0/Φ. At zeros of the function $L_n(\pi\Phi_0/\Phi) = 0$ (the flat band condition), the Landau bands have zero width. The Hofstadter butterfly is only defined for rational values of the flux quantum number $\Phi/\Phi_0 = p/q$, where p and q are integers. The number of subbands in the Hofstadter butterfly (and therefore in each LL) is q (see figure 1). We have to consider, however, that for even q the gap between miniband $m = q/2$ and $m+1$ vanishes, producing a pseudogap with vanishing density of states. Numerical calculations are usually restricted to all fractions with a maximum q_{max}. In figure 2, a fragment of the Landau bands in a finite energy belt around an arbitrarily chosen Fermi energy is shown. As seen in the lower panel of figure 2, the 6th Landau band is flat at the Fermi level, because the flat band condition $L_n(\pi\Phi_0/\Phi) = 0$ holds for this band at the chosen values of the parameter.

2.2. Magnetic oscillations in the diagonal conductivity

The energy spectrum alone cannot determine the diagonal conductivity that depends also on the mechanisms of the electron transport. Disorder not only broadens the minibands of the Hofstadter butterfly but also results in the localization. Numerical simulations show that in analogy with the unmodulated 2D conductors the extended states appear approximately at the center of minibands [23] and even the critical exponents are the same as in conventional IQHE [27]. These facts make it possible to apply the physics and results obtained for conventional IQHE to calculations of the diagonal conductivity in the Hofstadter problem. The localization is a key point because only extended states within the minibands contribute to the conductivity.

Qualitatively, the picture of conductivity at high magnetic fields is as follows. Electrons at the Landau orbitals drift along the equipotential contours of disorder potential and hop from one contour to another at places where these contours are close enough to make the hopping possible. For fixed disorder potential the configuration of the equipotential contours depends on the energy, making the electron percolation through the contours dependent on the Fermi energy. Numerical simulations show that for low disorder strength, when Landau bands do not overlap, only narrow energy stripes at the center of disordered bands are delocalized [8, 9], [20]–[22]. This fundamental fact makes it possible to simplify equations for the σ_{xx} in a weakly disordered 2D conductor in the perpendicular magnetic field because only extended states contribute to the diagonal conductivity [28, 29].

In a model in which the electron scattering time τ is independent of energy (the 'τ-approximation') and under the condition that temperature $T \gg \hbar/\tau$, the conductivity σ_{xx} can be written as follows [28, 29]:

$$\sigma_{xx}(B) \approx \sigma_\tau \frac{\hbar\omega_c}{2\pi T} \sum_{n,m} \cosh^{-2}\left(\frac{E_{n,m} - \mu}{2T}\right), \tag{4}$$

where

$$\sigma_\tau = \frac{e^2 N_L \tau \langle v_x^2 \rangle}{\hbar\omega_c} \tag{5}$$

Landau bands
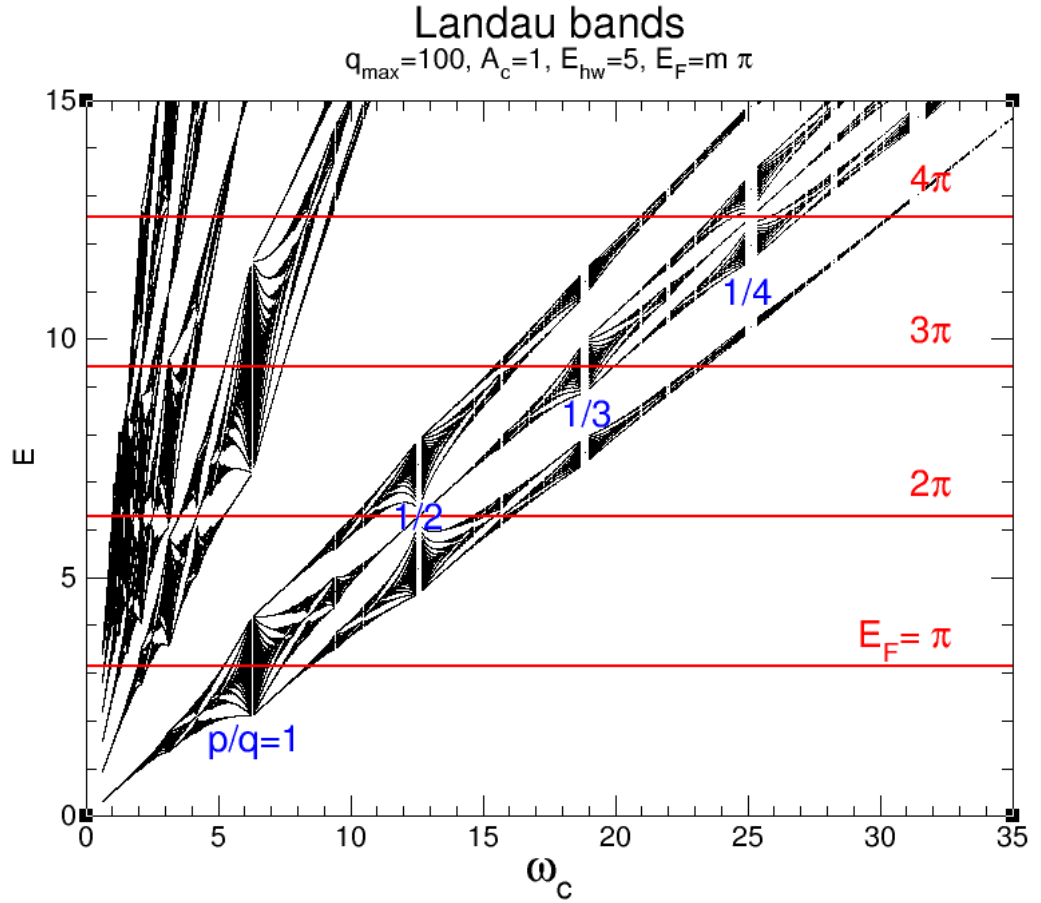$q_{max}=100$, $A_c=1$, $E_{hw}=5$, $E_F=m\,\pi$

Figure 1. Landau bands from equation (3) for the following parameters: $q_{max} = 100$, $A_c = 1$ is the unit cell area, $E_{hw} = V_0/2 = 5$ is the width of the Landau bands in the limit $B \to \infty$. The Fermi energy is chosen as $E_F = m\pi$ ($m = 1, 2, 3, 4$) in order to demonstrate a clear splitting of the peak from the highest Landau band into m subpeaks (see figure 3).

and the average of the squared velocity is given by

$$\langle v_x^2 \rangle = \frac{a^2}{2\hbar^2} \int_{\varepsilon_{min}}^{\varepsilon_{max}} d\varepsilon \, g(\varepsilon)|t_{\varepsilon,\varepsilon}|^2. \tag{6}$$

$E_{n,m}$ is the Landau spectrum (3), $t_{\varepsilon,\varepsilon}$ is the electron matrix element for hopping between the equipotential contours along which Landau orbitals are drifting in the perpendicular magnetic field, a is the average distance of hopping, $N_L = \Phi/S\Phi_0$ is the electron density at the LL, S is the sample area, and μ stands for the chemical potential. The shape of the density of states (DOS) within the Landau bands is assumed to be arbitrary and given by the function $g(\varepsilon)$. The integral in (6) is taken over the narrow stripes of delocalized states at the center of the Landau bands.

At low fields, $\hbar\omega_c \leqslant T$, equation (4) yields σ_{xx} as an oscillating function of $\hbar\omega_c$. Under the condition $\hbar\omega_c/T \gg 1$, the conductivity σ_{xx} becomes a sharply peaked function, which has maxima when one of the LLs exactly coincides with the chemical potential, $E_{n,m} = \mu$. The conductivity at maxima is equal to $\sigma_{xx} = \sigma_\tau \frac{\hbar\omega_c}{2\pi T}$. At minima, as well as at all other magnetic
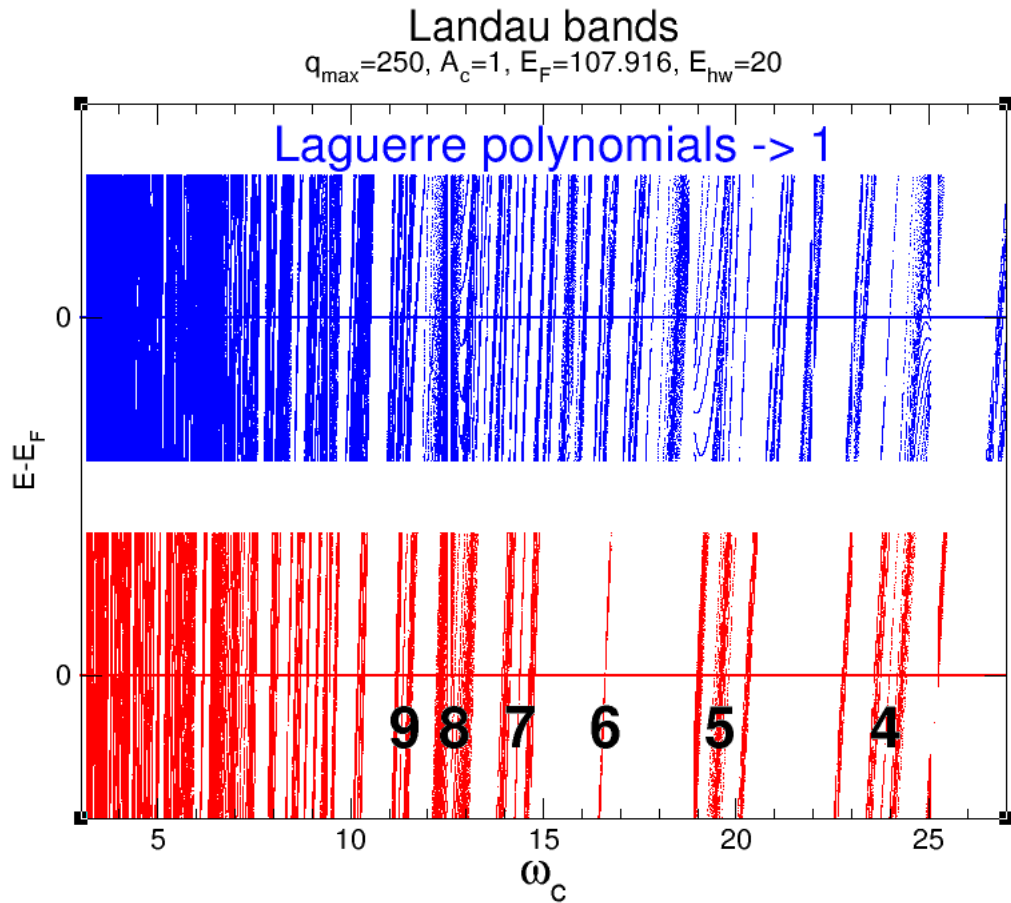
Figure 2. Landau bands around the Fermi level for some higher LLs (indices indicated by bold numbers) with and without modulation by the Laguerre polynomial factor. ($q_{max} = 250$, $A_c = 1$, $E_{hw} = 20$, $E_F = 106.916$. Notations are the same as in figure 1.)

fields for which the chemical potential μ falls between the LLs, the conductivity σ_{xx} becomes an exponentially small function of temperature and $\hbar\omega_c$:

$$\sigma_{xx} = \sigma_\tau \frac{\hbar\omega_c}{2\pi T} \exp\left(-\frac{|\hbar\omega_c - \delta\mu|}{T}\right), \tag{7}$$

where $\delta\mu$ is a separation between the chemical potential and the center of the nearest partially filled LL. The exponential smallness of the σ_{xx} in the Hall plateau regions is a manifestation of the localization of electrons elsewhere outside the narrow stripes of extended states. This point is crucial for the derivation of equation (4) [28, 29].

Before closing this section an important remark is in order. In the conventional IQHE regime, numerous experiments testify that the critical exponents of the metal-to-insulator transition and the shape of peaks in the diagonal conductivity σ_{xx} do not depend on the LL index n [30, 31]. In our approach this fact means that the factor σ_τ given by equation (5) is the same for all LLs. The latter is because the DOS $g(\varepsilon)$ and the averaged (over the random hopping matrix elements $|t_{\varepsilon,\varepsilon}|^2$) velocity squared $\langle v_x^2 \rangle$ can be taken as independent of the LL number.
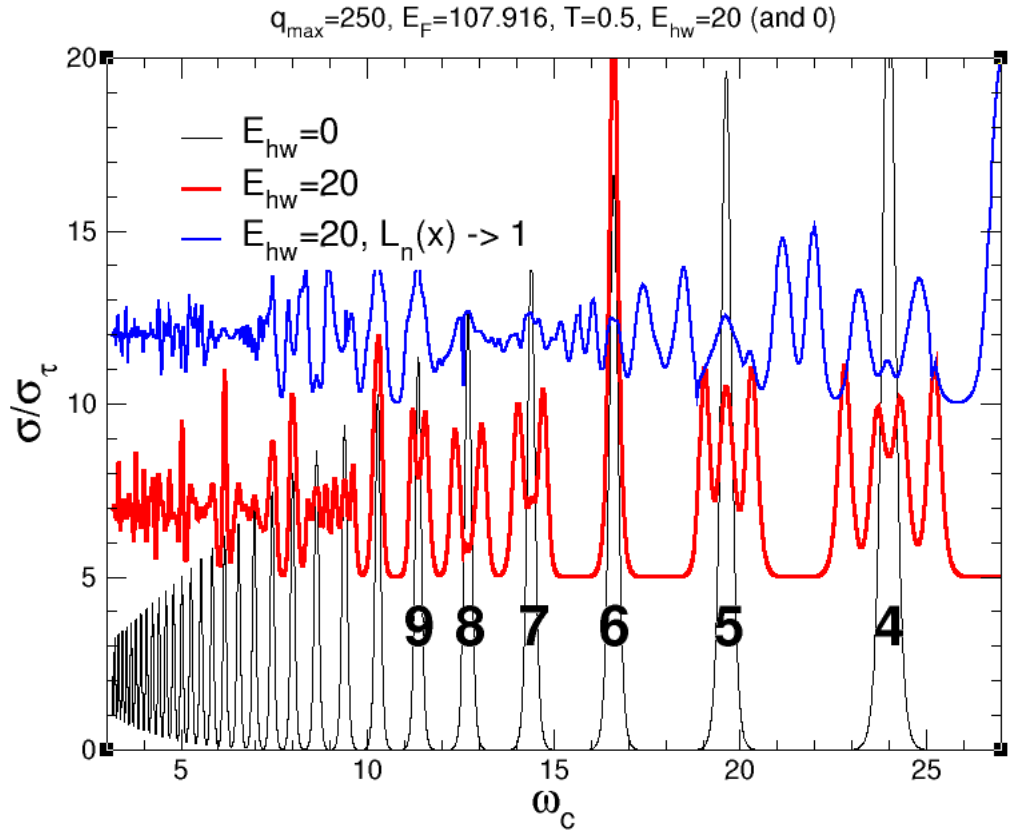
Figure 3. Diagonal conductivity for higher Landau bands as a function of the cyclotron frequency ω_c for $T = 0.5$. The parameters are the same as in figure 2, except for the half-width $E_{hw} = 0$ (bottom curve), $E_{hw} = 20$ (middle curve), $E_{hw} = 20$ and modulation factor $|L_n(\pi \Phi_0/\Phi)|$ replaced by unity (top curve). The LL index is indicated by bold numbers.

The situation changes in the Hofstadter spectrum in which LLs split into fractal minibands with the butterfly fine structure shown in figures 1 and 2. Under such conditions the quantities σ_τ in general may become dependent both on the LL and on miniband indices n, m. Because of that, the amplitudes within the bunches of the split peaks of the conductivity σ_{xx} may have different values. This effect is sample-dependent and cannot be described within any model of disorder. Having this in mind, we neglect for the sake of simplicity the dependence of the σ_τ on the indices n, m in equation (4). We will return to this point in the discussion of our results.

3. Numerical results

Numerical results for the quantum magnetic oscillations in the diagonal conductivity σ_{xx} based on equations (3)–(6) are shown figures 3 and 4. In figure 3 the influence of the internal miniband structure is demonstrated. The lowest curve corresponds to the case where the superlattice modulation is absent. In that case the diagonal conductivity displays the Shubnikov–de Haas (SdH) oscillations at low magnetic fields $\omega_c \ll 10$, which at higher fields $\omega_c > 10$ gradually cross over into a sequence of sharp peaks typical for the well-developed IQHE regime. Between
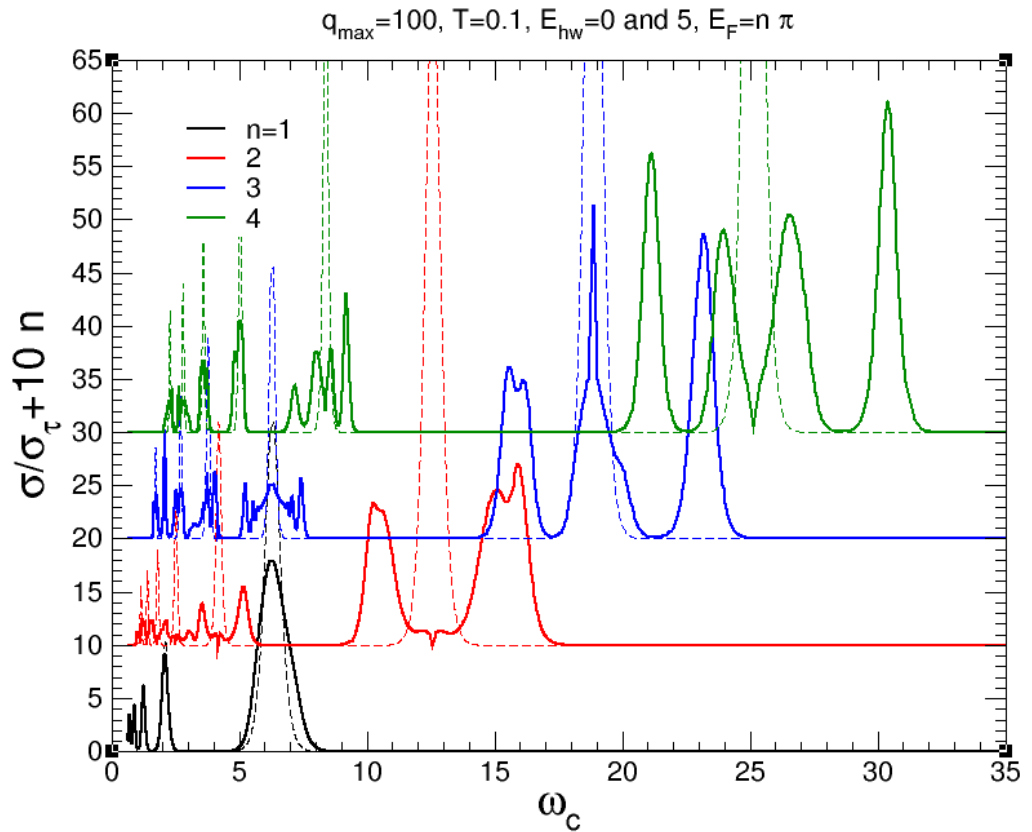
Figure 4. Diagonal conductivity as a function of cyclotron frequency ω_c for the Landau bands shown in figure 1. The Fermi energy is $E_F = n\pi$ ($n = 1, 2, 3, 4$ from bottom to top), and $T = 0.1$. The other parameters are the same as in figure 1, except that the curves for completely flat Landau bands with half width $E_{hw} = 0$ (broken curves) are included apart from the result corresponding to figure 1 with $E_{hw} = 5$ (full curves).

the peaks the diagonal conductivity σ_{xx} is exponentially small. Correspondingly, the Hall conductance σ_{xy} takes the quantized plateau values within these regions. The effect of the square superlattice modulation is illustrated by the middle graph in figure 3 and by plots in figure 4. One can see three basic distinctions from the conventional IQHE case. (i) The peaks in diagonal conductivity are split: (ii) the splitting is irregular and asymmetric because of the fractal nature of the Hofstadter energy spectrum; (iii) the gaps between the split Landau bands (and related Hall plateaus) become narrower with the superlattice modulation than without it. The number of peaks and their shapes within each split LL depend in a nontrivial way on the fractal Hofstadter miniband structure of this level at the Fermi energy E_F. For example, in figure 3 the peak with the Landau index $n = 6$ is not split at all, because this band is flat as shown in figure 2 for the same choice of the parameters. The peak at $\omega_c = 10$ is narrow enough so that the splitting is unresolved in the picture. The importance of the Laguerre polynomial factor $|L_n(\pi \Phi_0/\Phi)|$ for the shape of magnetic oscillations is illustrated by the top curve in figure 3 plotted for the same parameters as the lower curve, but without the Laguerre polynomial factor (it is replaced by unity, $|L_n(\pi \Phi_0/\Phi)| \to 1$ in calculations for this curve).

The Hofstadter miniband structure depends on the Fermi energy value as is clearly seen in figure 1. The corresponding effect on the conductivity is shown in figure 4 where the conductivity σ_{xx} is plotted for different values of E_F at lower temperatures than that adopted in figure 3. We see in figures 1 and 4 that the magnetic-field range of oscillations is shifted toward higher values with the increase of E_F. The fine structure of the peaks becomes more complex and the gaps between the split peaks grow wider for larger n.

In our numerical analysis we neglected variations of the chemical potential μ as a function of the magnetic field and replaced it by the Fermi energy E_F at zero magnetic field. Qualitatively, this is because all states within the sequence of the impurity-broadened LLs (described by the function $g(\varepsilon)$) are localized and fix the μ when thin stripes of extended states shift in the changing magnetic field. As we see, the assumption of constant μ agrees qualitatively with the experiment [18].

4. Summary and conclusions

In this paper we have demonstrated how the conventional IQHE oscillations of the diagonal conductivity do change under the influence of a weak square superlattice modulation. Motivated by recent experiments of the paper [18], we consider a regime in which disorder and temperature do not destroy completely the Hofstadter butterfly in the Landau bands. We first calculated the energy spectrum using equation (2). The corresponding miniband structure within the Landau fan is plotted in figures 1 and 2.

We then use equation (4) derived in [28, 29] for calculations of the σ_{xx} in the case of conventional IQHE under the condition $T \gg \hbar/\tau$. That condition (which assumes a weak disorder since the IQHE is a low-temperature phenomenon) simplifies the equation for the σ_{xx} but preserves all typical features in the shape of the diagonal conductivity in the IQHE regime. This is illustrated by the lowest curve in figure 3, which plots the σ_{xx} as a function of B in the absence of superlattice modulation. One can see that the curve picks up all the basic ingredients of the IQHE: sharp peaks and Hall plateaus (intervals where the conductivity σ_{xx} is exponentially small) at high fields and SdH oscillations in the low field region. If all states within the Landau band are localized, then the velocity squared (5), the amplitudes σ_τ, as well as the diagonal conductivity σ_{xx} are equal to zero. Thus, equation (4) gives a correct analytical description of the σ_{xx} in the IQHE regime.

We have shown that the square superlattice splits the peaks in diagonal conductivity in a quasi-regular fashion. Figures 3 and 4 demonstrate the relationship between the fractal LL splitting and the split-peak structures in the diagonal conductivity σ_{xx} under the conditions of the IQHE. The number of peaks in the conductivity $\sigma_{xx}(B)$ and their precise positions depend on the Hofstadter butterfly structure at the Fermi energy and magnetic field B as shown in figures 1 and 2. At the flat band condition $|L_n(\pi\Phi_0/\Phi)| = 0$, peaks in the function $\sigma_{xx}(B)$ are not split. Other peaks are split. The splitting, as one can see in figures 3 and 4, may be symmetric or not depending on the values of the Fermi energy and magnetic field. In reality, because equipotential contours are energy-dependent, the quantity σ_τ may vary from miniband to miniband making the amplitudes of the peaks in a bunch different. This effect is sample-dependent and cannot be considered in any specific model. Because of that, only a quantitative comparison with experiments makes sense.

Having this in mind we see that all the above mentioned features in the function $\sigma_{xx}(B)$ are in good qualitative agreement with the experimental observations of the paper [18]. In this paper,

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the resistivity was measured for large values of the Landau index under the conditions when Hall conductivity is quantized and is much larger in value than the diagonal one, $\sigma_{xy} \gg \sigma_{xx}$. This is clearly seen in figures 2 and 3 of the paper [18]. The resistivity $\rho_{xx}(B) = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{xy}^2)$ under these conditions is simply proportional to σ_{xx} and displays features similar to $\rho_{xx}(B)$ at every plateau-to-plateau transition region.

In conclusion, one remark is in order. A calculation of the function $g(\varepsilon)$ as well as a quantitative description of localization of electrons within this function is a very complex model-dependent problem which has not been solved so far. The strong point of our approach is that the main results can be obtained without precise knowledge of the form of the function $g(\varepsilon)$ because only a thin stripe of delocalized states contributes to the diagonal conductivity σ_{xx}. Details of the positions of extended states are also not necessary. This is important since after more than two decades of studies, the localization phenomenon still remains incompletely understood. Modern understanding of the localization phenomenon in the Landau bands in 2D conductors came mainly from the numerical analysis of different models and experiments. The current viewpoint on the problem is accumulated in the topological quantum Hall phase diagram in the plane disorder-magnetic field, which explains a transition from the scaling to non-scaling regime in the quantum Hall systems with the increase of disorder [20]. The scaling regime corresponds to the case where the width of extended state stripes goes to zero, while the non-scaling regime holds for stripes of finite sizes. The transition between these two regimes depends on the degree of the overlap of adjacent Landau bands and has a threshold on the disorder value that increases linearly with B. Below this critical value, only one delocalized state exists in each Landau band, which evolves with an increase of disorder into stripes of finite width merging into new bands at some (increasing linearly with B) value of disorder [20]. This scenario for the evolution of extended states with disorder is very similar to that established in the numerical simulations of the paper [23] in which the localization within the Hofstadter butterfly spectrum has been studied. It was found that extended levels within the subbands of the split LLs get closer as disorder increases, and contract into one at a certain $\hbar/\tau$. We used this result in our calculations assuming that the disorder is weak enough and less than the threshold value of [20] so that only one narrow stripe of delocalized states exists in each subband of the nonoverlapping Landau bands. These extended states do contribute, according to equation (4), to the peaks in diagonal conductivity σ_{xx} resolved in figures 3 and 4.

And now the last note. So far all theoretical papers on the IQHE in 2D systems with the Hofstadter butterfly spectrum have concentrated on calculations of the Hall conductivity σ_{xy} on the basis of equation (1), which yields only a thermodynamic part of the σ_{xy} [1, 2, 7, 23, 26]. In view of that, knowledge of the diagonal conductivity has additional value because the peaks in σ_{xx} determine the shape and the width of the transitional plateau-to-plateau regions in the Hall conductivity through the term σ_{xy}^C, which in the quasiclassical approximation equals $\sigma_{xy}^C = \omega_c \tau \sigma_{xx}$ [2].

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