Mathematisches Forschungsinstitut Oberwolfach

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# Topology

September 9th – September 15th, 2001

The Topology conference covered a wide variety of topics from algebraic topology and related fields ranging from low dimensional topology to homotopy theoretic questions. There were for example talks on orbifold cohomology, hyperbolic 3-manifolds, group actions, motivic cohomology, $L^2$-cohomology, knot theory and spaces of embeddings. A highlight was a series of three talks by P. Shalen with the title "Character varieties, 3-manifolds with cyclic $\pi_1$ and smallish knots", where among other things a program for proving the Poincaré conjecture was outlined.
The conference had about 50 participants from Europe, America and Australia. It was organized by Cameron Gordon (Austin), Wolfgang Lück (Münster) and Bob Oliver (Paris). There were roughly four one-hour talks per day to leave enough room for discussions and research. As always the staff of the institute did everything possible in order to help the participants concentrate on mathematics. Although this was harder than usual due to the cruel events on the 11th of September.

## Schedule

### Monday

| | | |
|---|---|---|
| 9:30 - 10.30 | P. Shalen | Character varieties, 3-manifolds with cyclic $\pi_1$, and smallish knots – Part I |
| 11:00 - 12:00 | C. Broto | The homotopy theory of fusion systems |
| 16:00 - 17:00 | O. Cornea | Hopf invariants, duality and periodic orbits of Hamiltonian flows |
| 17:15 - 18:15 | M. Weiss | Homotopy types of spaces of smooth embeddings |

### Tuesday

| | | |
|---|---|---|
| 9:30 - 10.30 | P. Teichner | Feynman diagrams and 3-dimensional gropes |
| 11:00 - 12:00 | M. Davis | $l^2$-cohomology of Artin groups |
| 16.00 - 17.00 | G. Powell | Cohomology operations in motivic cohomology |
| 17.15 - 18.15 | S. Boyer | Surgery on hyperbolic knots |

### Wednesday

| | | |
|---|---|---|
| 9:30 - 10.30 | Shalen, P. | Character varieties, 3-manifolds with cyclic $\pi_1$, and smallish knots – Part II |
| 11:00: - 12:00 | A. Adem | Aspects of orbifold cohomology and $K$-theory |
| 13.30 | | Excursion |

### Thursday

| | | |
|---|---|---|
| 9:30 - 10.30 | Rubinstein, J. | The Smale conjecture for lens spaces |
| 11:00 - 12:00 | Scharlemann, M. | Knots of tunnel one and genus one |
| 15.45 - 16:30 | Calegari, D. | Hyperbolic 3-manifolds and groups of homeomorphisms of $S^1$ |
| 16.40 - 17:25 | Leary, I. | Examples of groups of type VF |
| 17.35 - 18:20 | Whitehouse, S. | Bases for cooperations in $K$-theory |

### Friday

| | | |
|---|---|---|
| 9:30 - 10.30 | Shalen, P. | Character varieties, 3-manifolds with cyclic $\pi_1$, and smallish knots – Part III |
| 11:00 - 12:00 | Mahowald, M. | Construction of maps between spectra via elliptic curves |
| 16:00 - 17:00 | Ferry, S. | Pushing manifolds together |
| 17:15 - 18:15 | Browder, B. | Constructing finite group actions |

# Abstracts

### Aspects of Orbifold Cohomology and K-theory

Alejandro Adem

In this lecture we provide the basic definitions and properties of orbifold K-theory and orbifold cohomology. Using equivariant methods we exhibit a decomposition for K-theory and apply it to examples. We also describe how to generalize the string-theoretic Euler characteristic from global quotients to arbitrary reduced orbifolds. We also introduce a notion of twisted equivariant K-theory, using discrete torsion and calculate it in some cases. We apply this to the case of twisted symmetric products. We also discuss resolutions of orbifolds and how in some cases their ordinary K-theory should agree with the orbifold K-theory of the orbifold. This is joint work with Y.Ruan.

### Dehn fillings of large hyperbolic 3-manifold

Steven Boyer

In this talk I describe joint work with Cameron Gordon and Xingru Zhang on exceptional filling slopes of a compact, connected, orientable hyperbolic 3-manifold $M$ with one cusp. In this context, an exceptional Dehn filling slope is one for which the associated closed manifold is non-hyperbolic. It has been conjectured by Gordon that the distance (i.e. geometric intersection number) between two such slopes is at most 5 when the manifold is large, that is, when the manifold contains an essential closed surface. We verify this conjecture in various situations, for instance when the first Betti number of $M$ is at least 2. In order to do this we sharpen an inequality of Albert Fathi by showing that given a pseudo-Anosov mapping class $f$ of a closed, connected, orientable surface $S$ and an essential simple closed curve $\gamma$ in $S$, then the set of integers $n$ for which the composition $T_\gamma^n f$ is not pseudo-Anosov has diameter at most 5 (here $T_\gamma$ denotes a Dehn twist along $\gamma$).
For large manifolds $M$ of first Betti number 1 we obtain partial results. Let $S$ be a closed essential surface in $M$ and set

$$\mathcal{C}(S) = \{ \text{ slopes } r \mid \ker(\pi_1(S) \to \pi_1(M)) \neq \{1\}\}.$$

A *singular slope* for $S$ is a slope $r_0 \in \mathcal{C}(S)$ such that any other slope in $\mathcal{C}(S)$ is at most distance 1 from $r_0$. According to a result of Wu, singular slopes exist as long as $\mathcal{C}(S) \neq \emptyset$. We prove that the distance between two exceptional filling slopes is at most 5 if either
- there is a closed essential surface $S$ in $M$ with $\mathcal{C}(S)$ finite.
- there are singular slopes $r_1 \neq r_2$ for closed essential surfaces $S_1, S_2$ in $M$.

### The homotopy theory of fusion systems

Carles Broto

(joint work with Ran Levi and Bob Oliver)

A *fusion system* over a finite $p$-group $S$ is a category whose objects are the subgroups of $S$ and whose morphisms are monomorphisms of groups which include those induced by conjugation in $S$. A fusion system is *saturated* if it satisfies certain additional axioms formulated by Lluís Puig, and motivated by the properties of fusion within the subgroups of a given Sylow $p$-subgroup $S$ of a finite group $G$. A fusion system $\mathcal{F}$ over $S$ is rigidified by

a *centric linking system* which consists of a category $\mathcal{L}$ and a functor $\pi\colon \mathcal{L} \to \mathcal{F}$ which is injective on objects and surjective on morphisms. It also satisfies certain axioms motivated by the case of finite groups.
We define a *p-local finite group* as a triple $(S, \mathcal{F}, \mathcal{L})$, where $S$ is a finite $p$-group, $\mathcal{F}$ is a saturated fusion system over $S$, and $\mathcal{L}$ is a centric linking system associated to $\mathcal{F}$. Its classifying space is defined as the $p$-completion of the nerve of $\mathcal{L}$.
We show that the classifying space $|\mathcal{L}|_p^\wedge$ of a $p$-local finite group $(S, \mathcal{F}, \mathcal{L})$ has many of the same properties of the $p$-completed classifying space of a finite group. For example, $\pi_1(|\mathcal{L}|_p^\wedge)$ is a finite $p$-group and mapping spaces $\mathrm{map}(BQ, |\mathcal{L}|_p^\wedge)$, for finite $p$-groups $Q$, can be described in terms of homomorphisms from $Q$ to $S$ and of centralizers of their images. Also, $H^*(|\mathcal{L}|_p^\wedge; \mathbb{F}_p)$ is isomorphic to the subring of $H^*(BS; \mathbb{F}_p)$ of elements stable under fusion of $\mathcal{F}$.
Finally, we characterize the class of classifying spaces of $p$-local finite groups in the homotopy category of spaces. This class contains all $p$-completed classifying spaces of finite groups but it also contains exotic examples; that is, examples which do not come from a finite group.

## Constructing finite group actions

B. Browder

Let $G$ be a finite $p$-group, $U$ it's universal space. Let $Y$ be a free $G$-space, $S$ an $n$-dimensional $G$-space, and $f : S \times U \to Y$ a $G$-map.
Problem: Can one add free $G$-cells of dimension less or equal to $m$ ($m > n$) to get an $m$-dimensional $G$-space $X$ together with a map $F : X \times U \to Y$ such that

(i) $F$ induces isomorphism from $\pi_k(X \times U) \to \pi_k(Y)$ for $k < m$, and
(ii) $F_* : H_m((X \times U)/G; F_p) \to H_m(Y/G; F_p)$ is an isomorphism?

Theorem: Suppose $S$, $Y$ above 1-connected, $n > 2$. One can find $X$, $F$ as above if and only if the projection map

$$H_{m+1}(Y, X \times U; F_p) \to H_{m+1}(Y/G, (X \times U)/G; F_p)$$

is onto. One way to apply this, would be to construct $Y$ as an $m$-th stage Postnikov system, and try to find an $m$-dimensional $X$. The case where $S$ is empty leads to exotic actions of elementary abelian groups on spaces of the homotopy type of products of spheres.
One may also apply this to construct actions on familiar spaces with less familiar fixed point sets, or homotopy types.
One may apply this method to study the following question:
Problem: Given a free action of $G$ on $Y$, where $Y$ has the homotopy type of a finite complex, when is there a finite dimensional G space $X$, and a map $F : X \times U \to Y$ which is a mod p homology equivalence.
The work of Grodal and Smith gives an affirmative answer to this question in case Y has the homotopy type of the sphere. We can show that the question is equivalent to the question of whether the Borel-Quillen localization theorem hold for homotopy fixed points for subgroups, with certain restrictions on the isotropy subgroups.

**Hopf invariants, duality and periodic orbits of hamiltonian flows**

OCTAV CORNEA

Assume that $(M, \omega)$ is a fixed symplectic manifold. Suppose also that we fix a smooth function $H : M \to \mathbf{R}$. We can then associate to $H$ its hamiltonian vector field $X_H$ which is defined by requiring the equality $\omega(X_H, Y) = dH(Y)$ to hold for all vector fields $Y$.
A classical problem in non-linear analysis and geometry is to detect periodic orbits of the flow induced by $X_H$. Indeed, the origins of this problem can be traced back to Kepler and the two-body problem and after that to the work of Poincaré. More recentely, by work of Rabinowitz, Weinstein, Moser, Floer, Zehnder, Hofer and many other authors, this problem has been at the center of the development of symplectic topology. The standard, approach to this problem is via analysis on the free loop space of $M$. The homotopical properties of $M$ play little role in this approach as only the homology of $M$ actually appears. Essentially, all effective results take place in the case when $M$ is compact.

The purpose of this work[1] is to show that, in fact, homotopical properties related to the pair $(M, H)$ do have an important impact on the recurrence properties of $X_H$. This is best seen when $M$ is non-compact. Notice that in this case the existence of bounded orbits of $M$ is a first non-trivial step. Moreover, once bounded orbits are found by the $C^1$-closing lemma of Pugh and Robinson a generic perturbation will create periodic ones.

We therefore focus on the problem of existence of bounded orbits and we show that the non-vanishing of certain Hopf invariants associated to a *gradient* flow of $H$ leads to such existence results. This is proven by first identifying these Hopf invariants with certain bordism classes of connecting manifolds via Morse theory.
This approach is part of a more general program whose purpose is to understand and explicit the homotopical properties of flows. Other related results that fit in this more genral setting allow the detection of some non-smoothable Poincaré duality spaces and also provide obstructions to the existence of thickenings of certain $CW$-complexes in low codimension $\mathbf{R}^n$'s.

**The $\ell^2$-cohomology of Artin groups**

MICHAEL DAVIS
(joint work with Ian Leary)

For each Artin group $A$ we compute the reduced $\ell^2$-cohomology of the universal cover $\widetilde{X}$ of the "Salvetti complex", $X$. This is a CW-complex which is conjectured to be a model for the classifying space of the Artin group. In the many cases when this conjecture is known to hold our calculation describes the reduced $\ell^2$-cohomology of the Artin group. If $L$ is the nerve of the associated Coxeter group and $CL$ denotes the cone on $L$, then the answer is that the reduced $\ell^2$-cohomology of $\widetilde{X}$ is $H^*(CL, L) \otimes \ell^2(A)$.

[1] Preprints: Homotopical Dynamics I,II,III,IV at:
http://www-gat.univ-lille1.fr/~cornea/octav.html
I - Erg. Th.& Dyn. Syst. 2 (2000), III - Duke Math. J. 209 (2001)

**Pushing manifolds together**

Steven C. Ferry
(joint work with A.N. Dranishnikov)

**Definitions.**
*(*i.) If $X$ and $Y$ are compact subsets of a metric space $Z$, we say that $d_Z(X, Y) < \epsilon$ if every $\epsilon$-neighborhood of $X$ in $Z$ contains $Y$ and every $\epsilon$-neighborhood of $Y$ contains $X$. This defines the *Hausdorff metric* on compact subset of $Z$.

*(*ii.) If $X$ and $Y$ are compact metric spaces, we define $d_{GH}(X, Y) < \epsilon$ if there is a metric space $Z$ containing isometric copies of $X$ and $Y$ so that $D_Z(X, Y) < \epsilon$. This defines the *Gromov-Hausdorff metric* on isometry classes of compact metric spaces. We will denote this metric space by $GH$.

*(*iii.) If $\mathcal{C}$ is a subset of $GH$, a continuous, monotone function $\rho : [0, R) \to [0, \infty)$ is a *contractibility function* for $\mathcal{C}$ if for every $X \in \mathcal{C}$ and $x \in X$, and $t \in [0, R)$, the ball $B_t(x)$ contracts to a point in $B_{\rho(t)}$.

It is not hard to see that if $\rho : [0, R) \to [0, \infty)$ is a contractibility function, then there is an $\epsilon > 0$ so that if $M$ and $N$ are $n$-manifolds with contractibility function $\rho$ and $d_{GH}(M, N) < \epsilon$, then $M$ and $N$ are homotopy equivalent. This leads to notions of deformation and rigidity in a class $\mathcal{C} \subset GH$.

**Definitions.**
*(*iv.) A *deformation* of a manifold $M$ in $\mathcal{C}$ is a map $\alpha : (0, 1] \to \mathcal{C}$ so that $\alpha(t) \in \mathcal{C}$ for $t > 0$, $\alpha(1) = M$, and so that $\lim_{t\to 0} \alpha(t)$ exists.

*(*v.) We will say that $M$ *deforms to* $N$ *in* $\mathcal{C}$ if there are deformations $\alpha$ and $\beta$ of $M$ and $N$ so that $\lim_{t\to 0} \alpha(t) = \lim_{t\to 0} \beta(t)$. Notice that $\alpha$ and $\beta$ give a preferred homotopy equivalence $M \to N$. We will say that such a homotopy equivalence is *realized by a deformation in* $\mathcal{C}$.

**Theorem.**
Let $n \geq 7$. There is a map $H_{n+1}(M^{[2]}, M; \mathbb{L}) \to \mathcal{S}(M)$ so that $[f] \in \mathcal{S}(M)$ is realized by a deformation if and only if $[f]$ lifts to an odd torsion element in $H_{n+1}(M^{[2]}, M; \mathbb{L})$. Here, $M^{[2]}$ is the second stage of the Postnikov tower of $M$ and $\mathcal{S}(M)$ is a group (the *structure set of* $M$?) whose elements are equivalence classes of homotopy equivalences from closed $n$-manifolds to $M$.

**Definition.** We say that $M$ is *rigid* if every homotopy equivalence $f : N \to M$ which is realized by a deformation is homotopic to a homeomorphism.

**Corollary.** Closed aspherical manifolds and complex projective spaces are rigid, while if $L$ is a lens space with odd order fundamental group and $f : L' \to L$ is a homotopy equivalence, then $f \times id : L' \times S^{2\ell+1} \to L \times S^{2\ell+1}$ is realized by a deformation, provided that the dimension of $L \times S^{2\ell+1}$ is at least 7.

## Some examples of groups of type VF

Ian J. Leary

(joint work with Brita Nucinkis)

Let $G$ be a discrete group. A model for $\underline{E}G$ is a $G$-CW-complex such that all cell stabilizers are finite, and such that the fixed point subcomplex for any finite subgroup is contractible. A group $H$ is of type F if there is a finite model for $EH$, and a group $G$ is of type VF if it contains a finite-index subgroup of type F. We construct groups $G$ of type VF having any of the following properties:

(1) There is no finite-type model for $\underline{E}G$;

(2) The minimal dimension of a model for $\underline{E}G$ is strictly greater than the virtual cohomological dimension of $G$;

(3) $G$ contains infinitely many conjugacy classes of elements of finite order.

For a finite flag complex $L$, the right-angled Artin group $A_L$ has generators the vertices of $L$ subject only to the relations that the ends of each edge commute. The group $B_L$ is the kernel of the homomorphism from $A_L$ to $\mathbb{Z}$ that sends each generator to 1. M. Bestvina and N. Brady showed that $B_L$ is of type F if and only if $L$ is contractible.

Our groups are constructed as semi-direct products $B_L : Q$, where $Q$ is a group of automorphisms of the flag complex $L$. Property (1) holds for this group whenever $L$ is contractible but not $Q$-equivariantly contractible. Property (2) holds whenever $L$ is contractible, and contains a top-dimensional simplex in a free $Q$-orbit whose boundary consists of simplices in non-free $Q$-orbits. The construction of examples having property (3) is more complicated, and involves infinite flag complexes.

## Construction of maps between spectra via elliptic curves

Mark Mahowald

This talk was a report on an aspect of a joint project with Paul Goerss, Hans-Werner Henn and Charles Rezk. (Part of the work was done in a RIP at Oberwolfach last year.) The object of the project is to construct a resolution of spectra which gives a presentation of the $K(2)$ local sphere. The basis of the resolution is a resolution of the Morava stabilizer group by modules extended from finite subgroups. The talk concentrated on the construction of the first map which can be viewed as coming from an isogeny between two ellptic curves of degree 3 (at the prime 2).

## Cohomology operations in motivic cohomology

Geoffrey M.L. Powell

Motivic cohomology with $\mathbb{Z}/2$-coefficients is a bigraded cohomology theory which is defined on the Morel-Voevodsky $\mathbb{A}^1$-local homotopy category with respect to the Nisnevich topology on the category of smooth schemes over a field $k$ of characteristic zero.

Steenrod squaring operations, $\mathbb{S}\mathrm{q}^i$, for motivic cohomology were constructed by Voevodsky using a version of the quadratic construction. The programme to prove that the motivic Steenrod algebra of bistable cohomology operations is generated as an algebra over the coefficient ring by the $\mathbb{S}\mathrm{q}^i$ is presented, based on calculating the (stable) motivic cohomology of the representing motivic Eilenberg-MacLane spaces.

The Suslin-Voevodsky algebraic Dold-Thom theorem and the $\mathbb{A}^1$-local version of group completion relate the motivic Eilenberg-MacLane spaces to symmetric products of certain schemes; two technical points arise: the 'sphere' $S^{2n,n}$ is not represented by a scheme but by the quotient sheaf $\mathbb{P}^n/\mathbb{P}^{n-1}$ and the symmetric products are in general not smooth.
The programme involves the calculation of the motivic cohomology with compact supports of symmetric products of affine space $\mathbb{A}^n$. Transfers are used to reduce to (products of) iterates of the symmetric square functor. The calculation is carried out by adapting techniques of Totaro for considering a Borel-Moore cohomology theory of a symmetric square.

### The Smale conjecture for lens spaces

J. Rubinstein

Hatcher in 1983 proved that the space of diffeomorphisms of the 3-sphere is homotopy equivalent to the orthogonal group $O(4)$, as conjectured by Smale in 1959. A method is outlined to show a similar result for all lens spaces other than $S^3$ or $RP^3$, namely $Diff$ is homotopy equivalent to $Isom$, the finite Lie group of isometries. The main idea is to use the methods of Scharlemann and the author to compare any two height functions on a lens space and to show then that a finite parametrised family of such height functions can be homotoped into either a fixed height function or a fixed family of vertical height functions, in a similar manner to work of McCullough and the author.

### There are no unexpected tunnel number one knots of genus one

Martin Scharlemann

We show that the only knots that are tunnel number one and genus one are those that are already known: 2-bridge knots obtained by plumbing together two unknotted annuli and the satellite examples classified by Eudave-Munoz and by Morimoto-Sakuma.
The principal new tools are a useful way of defining width for a 3-valent graph in $S^3$ and a controlled way of loading the knot onto a neighborhood of such a graph. We analyze how the knot loading allows the graph to be thinnned and show that eventually either the graph contains an unknot or the knot tunnel can be pushed onto the Seifert surface. In either of these circumstances the result (known as the Goda-Teragaito Conjecture) was already known.

### Title: Character varieties, 3-manifolds with cyclic $\pi_1$, and smallish knots, Part I—III

P. Shalen

Joint work of mine with Marc Culler, based on the study of $\mathrm{SL}_2$-character varieties of hyperbolic knot and link groups, shows that if $\Sigma$ is a closed 3-manifold with cyclic fundamental group, there are restrictions on the set of essential surfaces in the exterior of a knot in $\Sigma$ that do not hold for knots in an arbitrary closed 3-manifold. This leads to a program for proving the Poincaré Conjecture, or more generally the conjecture that 3-manifolds with finite cyclic $\pi_1$ are lens spaces, which involves showing that general non-Haken closed 3-manifolds contain knots with certain restrictions on the set of essential surfaces in their exteriors. One aspect of this program concerns the existence of knots that are "smallish" in the sense that their exteriors contain no bounded essential surfaces whose boundary curves are meridians. An ongoing joint project of mine with Culler, Nathan Dunfield and William Jaco is devoted to the question of existence of smallish knots. The approach is based on the idea of looking for an edge in a one-vertex triangulation—more precisely, a 0-efficient triangulation in the sense of Jaco and Rubinstein—which defines a smallish knot. We have shown that when a self-adjacent edge $e$ in such a triangulation defines a non-smallish knot, one can associate with $e$ a certain kind of polyhedron in the manifold, called a veeblefetzer, which can be "crushed" in a way similar to the way in which Jaco and Rubinstein crushed 2-sphere in producing 0-efficient triangulations. We hope that by iterating the crushing process we will be able to find either a smallish knot or a triangulation with no self-adjacent edge. We would also like to generalize this in order to show that in any non-Haken manifold one can find either a smallish knot or a triangulation satisfying certain local restrictions which might imply that $\pi_1$ is infinite.

### Feynman diagrams and 3-dimensional gropes

Peter Teichner

We explain the notion of a *grope cobordism* between two knots in a 3-manifold. Each grope cobordism has a type that can be described by a rooted unitrivalent tree. By filtering these trees in different ways, we show how the Goussarov-Habiro approach to finite type invariants of knots is closely related to our notion of grope cobordism. Thus our results can be viewed as a geometric interpretation of finite type invariants.
An interesting refinement we study are knots modulo *symmetric* grope cobordism in 3-space. On one hand this theory maps onto the usual Vassiliev theory and on the other hand it maps onto the Cochran-Orr-Teichner filtration of the knot concordance group, via symmetric grope cobordism in 4-space. In particular, the graded theory contains information on finite type invariants (with degree $h$ terms mapping to Vassiliev degree $2^h$), Blanchfield forms or S-equivalence at $h = 2$, Casson-Gordon invariants at $h = 3$, and for $h = 4$ one has the new von Neumann signatures of a knot.

### Homotopy types of spaces of smooth embeddings

Michael Weiss
(joint work with Tom Goodwillie and John Klein)

Let $M^m$ and $N^n$ be smooth manifolds, without boundary for simplicity, $n-m \geq 3$. Drawing inspiration from an old result due to Haefliger, we describe the homotopy type of the space of smooth embeddings $\mathrm{emb}(M, N)$ in terms of spaces of equivariant and stratified smooth maps from powers of $M$ to powers of $N$.
*Definition.* Let $R \subset S$ be finite sets. A smooth map $f\colon M^S \to N^R$ is *weakly stratified* if, for every equivalence relation $\eta$ on $R$ we have

$$f^{-1}(N^{R/\eta}) = M^{S/\eta},$$

and *stratified* if both $f$ and $Tf\colon (TM)^S \to (TN)^R$ are weakly stratified. (Note that $N^{R/\eta} \subset N^R$ and $M^{S/\eta} \subset M^S$.) Denote the space of these maps by

$$\mathrm{strat}(M^S, N^R).$$

The construction $\mathrm{strat}(M^S, N^R)$ is contravariantly functorial in $R$: an inclusion $R' \to R$ induces a projection $N^R \to N^{R'}$ which can be composed with stratified maps from $M^S$ to $N^R$. The construction $\mathrm{strat}(M^S, N^R)$ is covariantly functorial in $S$: an inclusion $S \to S'$

induces a projection $M^{S'}$ to $M^S$ which can be composed with stratified maps from $M^S$ to $N^R$.

Let $\Theta_k(M,N)$ be the space of fixed points of the obvious action of $\Sigma_k$ on the homotopy limit of the functor $(R,S) \mapsto \mathrm{strat}(M^S, N^R)$ where $R$ and $S$ run through the subsets of $\{1,2,3,\ldots,k\}$ and $R \subset S$. Assume $k \geq 2$. Since every smooth embedding $e\colon M \to N$ determines stratified maps $M^S \to N^R$ for all $S$ and $R \subset S$ by coordinate–wise application of $e$, we have a comparison map

$$\mathrm{emb}(M,N) \to \Theta_k(M,N).$$

*Theorem.* This map is $(1-m+(k+1)(n-m-2)$–connected (and since we are assuming $n-m-2>0$, the connectivity tends to $\infty$ as $k \to \infty$).

The special case $k=2$ is a mild reformulation of a result due to Haefliger (1961/62) and Dax (1973). The homotopy limit in the theorem has the following very explicit description: It is the space of natural transformations from $(R,S) \mapsto [0,1]^{S\setminus R}$ to $(R,S) \mapsto \mathrm{strat}(M^S, N^R)$, where $R \subset S \subset \{1,2,\ldots,k\}$ as before. Here we identify $[0,1]^{S\setminus R}$ with the space of maps from $\{1,2,3,\ldots,k\}$ to $[0,1]$ which take $R$ to 0 and the complement of $S$ to 1.

The proof of the theorem is fairly straightforward from the main results of embedding calculus, but of course these results rely on difficult multiple disjunction theorems (beginning with Morlet disjunction, then Goodwillie's thesis, and more recent work by Goodwillie and Goodwillie–Klein). The paper will appear in "Topology".

## Bases for cooperations in $K$-theory

*Sarah Whitehouse*

(joint work with Francis Clarke and Martin Crossley)

Gaussian polynomials are used to define bases with good multiplicative properties for the algebra $K_*(K)$ of cooperations in complex $K$-theory and for the invariants under conjugation.

The strategy is to first work $p$-locally and then globalise to integral results. For example, here is the $p$-local result for odd $p$.

**Theorem** Let $p$ be an odd prime and choose $q$ primitive modulo $p^2$. Let the polynomials $f_n(w) \in \mathbb{Q}[w]$ be given by

$$f_n(w) = \prod_{i=0}^{n-1} \frac{w-q^i}{q^n-q^i}.$$

Then $\{w^{-\lfloor n/2 \rfloor} f_n(w) : n \geq 0\}$ is a basis for $K_0(K)_{(p)}$.

The bases can be used to obtain a more explicit form of a theorem of Keith Johnson which characterises operations in $K$-theory in terms of their action on the coefficient groups.

We have similar results for the Adams summand, for $KO$ for $p=2$ and for connective versions of the various theories. The duals of our basis elements are $K$-theory operations which we can describe explicitly in terms of Adams operations. We hope to use our results to obtain new insight into the ring structure of the operations.

*Edited by Holger Reich*

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