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Asymptotically stable compensation of soliton self-frequency shift

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Abstract

We report the cancellation of the soliton self-frequency shift in nonlinear optical fibers. A soliton which interacts with a group velocity matched low intensity dispersive pump pulse, experiences a continuous blue-shift in frequency, which counteracts the soliton self-frequency shift due to Raman scattering. The soliton self-frequency shift can be fully compensated by a suitably prepared dispersive wave. We quantify this kind of soliton-dispersive wave interaction by an adiabatic approach and demonstrate that the compensation is stable in agreement with numerical simulations.

1 Introduction

A soliton propagating along a nonlinear optical fiber is subject to Raman scattering, the "red end" of the soliton spectrum is amplified at the expense of the "blue end" [1]. This results in what is called the soliton self-frequency shift (SSFS), a noticeable decrease of the carrier frequency of subpicosecond solitons [2]. The Raman scattering plays an important role, e.g., in optical supercontinuum [3, 4], but in soliton communications systems it is desirable to cancel its effect. The SSFS depends strongly on pulse-width [5], so solitons with initially equal carrier frequency but slightly different durations will gradually get different carrier frequencies (velocities) under Raman scattering, which results in jitter [6].

SSFS compensation may be mediated by cross-phase modulation (XPM) between the soliton and an accompanying dispersive wave (DW) [7]. The latter appears naturally if a pulse whose initial power exceeds that of a soliton with the same duration, brakes down into just that soliton and a DW [2]. Initially both, soliton and DW, have the same carrier frequency. Other possibilities for SSFS compensation include, e.g., bandwidth-limited amplification [8], pump by an additional soliton [9], interplay between Raman effect and Cherenkov radiation [10, 11]. In this Letter we demonstrate how the SSFS can be counteracted by XPM interaction with a prearranged pump DW, which (i) is much weaker than the soliton and (ii) has a distinctly different carrier frequency. Soliton collision with chirped DWs for SSFS compensation has been suggested in [12] based on numerical results.

A typical interaction of this kind is seen in Fig. 1. The electromagnetic power density [Fig. 1(a)] is plotted in space-time domain in a frame that co-moves with the unperturbed soliton. The DW approaches the initially stationary soliton (zero delay) and, being almost perfectly reflected [13, 14], yields an interference picture seen to the left of the soliton. The latter is deflected [red line in Fig. 1(a)] and compressed [red line in Fig. 1(b)]. The reflected part of the DW is frequency shifted, as seen in the frequency domain [Fig. 1(d)]. The soliton frequency is also shifted during reflection [Fig. 1(c)]. Our numerical simulations use a generalized nonlinear Schrödinger equation (GNLSE) with Raman term [4]. Without the DW, both the soliton's frequency and amplitude degrade, whereas its trajectory is deflected towards larger delays (sole Raman effect, not shown). This degradation is completely compensated by the DW after an initial transient phase, moreover, we will see below that the compensation is stable.

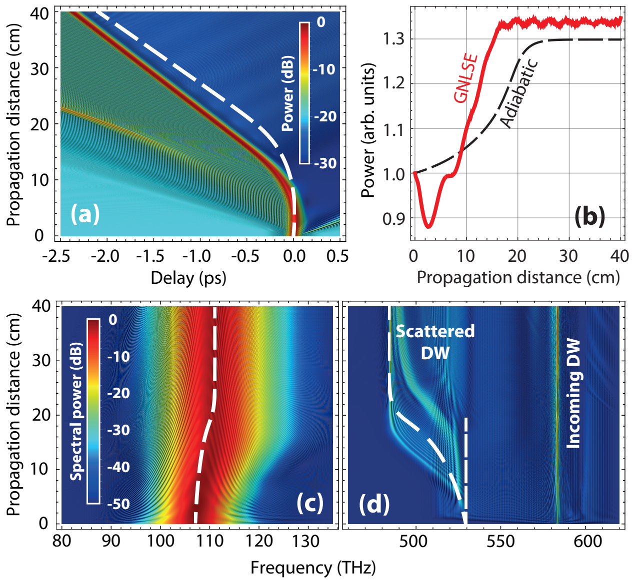

Figure 1: GNLSE calculation of a DW at 3.67 PHz interacting with a soliton at 0.67 PHz. (a) Power in space-time domain. The white dashed line is the soliton trajectory in the adiabatic approximation. (b) Soliton peak power from GNLSE (red line) and adiabatic theory (dashed line). (c,d) spectral power for soliton and DW respectively. Dashed lines are from adiabatic equations (7–9).

Dispersion properties of bulk silica meet all requirements for the interaction. Fig. 2 shows frequency dependencies of group velocity β' and group velocity dispersion (GVD) β''. The initial soliton carrier frequency ω_a and DW frequency $\omega_b + \Omega$ are chosen from opposite sides of the zero dispersion frequency ($\beta''(\omega_{\text{ZDF}}) = 0$) such that they provide almost equal group velocities. The reference frequency ω_b corresponds to the equal group velocities, $\beta'(\omega_a) = \beta'(\omega_b)$. In the moving frame we deal with the common delay $\tau = t - \beta'(\omega_a)z = t - \beta'(\omega_b)z$. The propagation distance z is measured along the fiber, t is the physical time variable. The frequency offset Ω should lie in a small interval around ω_b (shaded gray in Fig. 2) in order for the two pulses to interact [14]. A nearly perfect DW reflection (Fig. 1a) can be understood as a fiber-optical analog of the event horizon [15] or with quantum mechanical scattering theory [16].

In what follows, we quantify XPM interactions, like the one shown in Fig. 1. The standard soliton perturbation theory [17] was adapted to the problem at hand in [16]. It yields an adiabatic description of how a soliton evolves when colliding with a DW. Here we extend the theory to include the influence of Raman scattering, as outlined in the next section. Further we demonstrate how this description suggests that a stable compensation of the SSFS can occur. We describe a simple procedure to specify DW parameters that will produce cancellation of the SSFS. The last section deals with the stability analysis, numerical tests, and discussion.

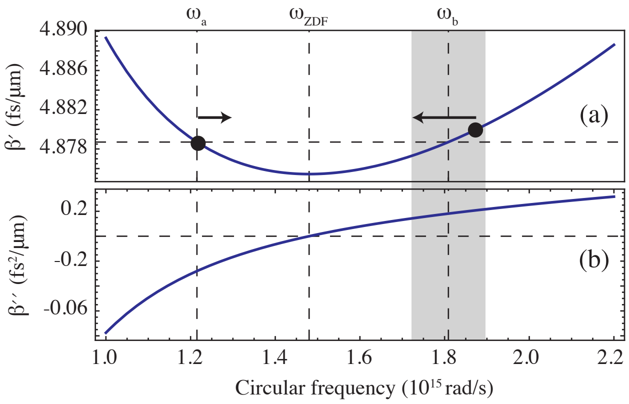

Figure 2: A typical profile (bulk silica) of (a) the group delay $\beta'(\omega)$ and (b) GVD $\beta''(\omega)$ that leads to the collision phenomenon shown in Fig. 1. Collision can only be realized for initial DW frequency offsets in a small interval (shaded gray) around the reference frequency of matching group velocity. Initial frequencies of soliton and DW are indicated by bullets, frequency shifts are indicated by arrows.

2 Model equations

An adiabatic description of interactions, like in Fig. 1, is derived from two XPM-coupled GNLSEs, one centered at ω_a for the soliton envelope $\psi(z,\tau)$, and one centered at ω_b for the DW envelope $\psi_b(z,\tau)$. The DW equation is simplified such that it can be solved as a problem of plane wave scattering at a moving "solitonic" potential barrier. The soliton equation is reformulated so that all higher-order terms in the GNLSE are treated as perturbations

$$\begin{aligned} &i\partial_z\psi + i\left[\beta'(\omega_a+\nu) - \beta'(\omega_a)\right]\partial_\tau\psi \\ &\quad - \frac{\beta''(\omega_a+\nu)}{2}\partial_\tau^2\psi + n_{2a}\frac{\omega_a+\nu}{c}\left|\psi\right|^2\psi = R\left(\psi,\psi_b\right). \end{aligned} \tag{1}$$

The most interesting point about this perturbation equation is that it explicitly takes into account a varying soliton carrier frequency $\omega_a+\nu$, where $\nu = \nu(z)$, $\nu(0) = 0$ is the yet unknown detuning from the initial carrier frequency ω_a. Especially the GVD $\beta''(\omega)$ and the nonlinear term on the left-hand side of (1) follow the soliton frequency shift. Therefore also the deviation from the initial group delay $\beta'(\omega_a)$ appears in the equation. All higher-order terms are contained in the perturbation $R\left(\psi,\psi_b\right)$

$$\begin{aligned} R(\psi,\psi_b) =& -\sum_{m=3}^{M}\frac{\beta^{(m)}(\omega_a+\nu)}{m!}\left[i\partial_\tau\right]^m\psi - \tau\frac{d\nu}{dz}\psi \\ &- \frac{2n_{2a}}{c}[\omega_a+\nu+i\partial_\tau]\left[\left|\psi_b\right|^2\psi\right] - \frac{n_{2a}}{c}i\partial_\tau\left[\left|\psi\right|^2\psi\right] \\ &- \frac{f_R n_{2a}}{c}\left[\omega_a+\nu+i\partial_\tau\right]\left[\mathcal{I}(\psi)\psi\right], \\ \mathcal{I}(\psi) =& \int_{-\infty}^{\tau} h(\tau-\tau')\left|\psi(z,\tau')\right|^2 d\tau' - \left|\psi(z,\tau)\right|^2. \end{aligned} \tag{2}$$

The terms with derivatives of the wave vector $\beta(\omega)$ account for higher-order dispersion, the first term in the second line describes XPM, the second is the self-steepening term. The perturbation term containing $d\nu/dz$ results from the necessary reformulations [16] of the standard soliton perturbation equation (as found for example in [17]) in order to include ν in the GVD. The term in the third line describes Raman scattering. The included Raman response function for fused silica reads

$$h(\tau) = \frac{\nu_1^2 + \nu_2^2}{\nu_1} e^{-\nu_2 \tau} \sin \nu_1 \tau, \tag{3}$$

and we used parameters $f_R = 0.18$, $1/\nu_1 = 12.2$ fs, and $1/\nu_2 = 32$ fs [18].

The fundamental soliton solution of the unperturbed (1) reads in a general formulation

$$\psi = \frac{1}{\sigma} \sqrt{\frac{|\beta^{(2)}(\omega_a + \nu)|c}{[\omega_a + \nu] n_a}} \frac{e^{\Theta}}{\cosh \frac{1}{\sigma}[\tau - \mathcal{T}]}, \tag{4}$$

with frequency offset $\nu = \nu(z)$ from the initial carrier frequency ω_a, duration $\sigma = \sigma(z)$, temporal delay $\mathcal{T} = \mathcal{T}(z)$, and a phase $\Theta = \Theta(z)$. It is assumed that all soliton parameters are z-dependent and slowly evolve under the influence of the DW. The soliton perturbation theory provides ODEs for these parameters after $|\psi_b|^2$ is specified. The initial parameter values are denoted by $\sigma(0) = \sigma_a$, etc.

The DW equation was derived by a series of simplifications applied to a full GNLSE. It was linearized with respect to the DW envelope ψ_b, also higher-order effects (dispersion, self-steepening, and Raman scattering) have no strong influences on a plane DW, and are neglected accordingly. The resulting equation reads

$$i\partial_z \psi_b - \frac{\beta''(\omega_b)}{2} \partial_\tau^2 \psi_b + n_{2b} \frac{2\omega_b}{c} |\psi|^2 \psi_b = 0. \tag{5}$$

It is mathematically equivalent to the quantum mechanical Schrödinger equation for a scattering problem: a plane wave with the power P_b is reflected at a moving barrier with a hyperbolic secant shape. It can be solved analytically [19]:

$$|\psi_b|^2 = P_b \mathfrak{T} \left| F\left(\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \frac{1 - \tanh \frac{\tau - \mathcal{T}}{\sigma}}{2}\right) \right|^2, \tag{6}$$

where F is a Gaussian hypergeometric function, the quantity

$$\mathfrak{T} = \frac{\sinh^2(\pi \bar{\Omega} \sigma)}{\cosh^2(\pi s) + \sinh^2(\pi \bar{\Omega} \sigma)},$$

is the transmission coefficient of the scattering problem, both using parameters

$$\mathfrak{a}, \mathfrak{b} = \frac{1}{2} - i\bar{\Omega}\sigma \pm is, \quad \mathfrak{c} = \frac{1}{2} - i\bar{\Omega}\sigma, \quad \bar{\Omega} = \Omega - \frac{\mathcal{B}}{\beta''(\omega_b)},$$

$$\mathcal{B} = \beta'(\omega_a + \nu) - \beta'(\omega_a) - \frac{\beta''(\omega_a + \nu)}{[\omega_a + \nu]\sigma^2} + \frac{\beta'''(\omega_a + \nu)}{6\sigma^2} + \cdots,$$

$$s = \frac{1}{2} \sqrt{16 \frac{|\beta''(\omega_a + \nu)|}{\beta_b''} \frac{\omega_b}{\omega_a + \nu} \frac{n_{2b}}{n_{2a}} - 1}.$$

Now, $|\psi_b|^2$ from (6) is inserted into (2) and then (1) is used to derive ODEs for the soliton parameters $\sigma(z)$, $\nu(z)$, and $\mathcal{T}(z)$. The phase $\Theta(z)$ can safely be ignored. The soliton duration $\sigma(z)$ happens to be explicitly expressed in terms of $\nu(z)$

$$\sigma(\nu) = \frac{\beta''(\omega_a+\nu)}{\beta''(\omega_a)} \frac{\sigma_a}{\left[1+\frac{\nu}{\omega_a}\right]^2}. \tag{7}$$

The ODE for the soliton frequency offset reads

$$\begin{aligned} \frac{d\nu}{dz} &= -\frac{\pi f_R}{4}\sigma\beta''(\omega_a+\nu)\mathcal{R}_1(\nu) \\ &+ \frac{4\mu\mathfrak{T}}{\sigma L_a}\left[1+\frac{\nu}{\omega_a}\right]\int_0^1 d\zeta\,|F(\mathfrak{a},\mathfrak{b},\mathfrak{c},\zeta)|^2\,[2\zeta-1]\,, \end{aligned} \tag{8}$$

and for the soliton delay we get

$$\begin{aligned} \frac{d\mathcal{T}}{dz} &= \mathcal{B} - \frac{3\pi f_R}{8}\frac{\sigma\beta^{(2)}(\omega_a+\nu)}{\omega_a+\nu}\mathcal{R}_2(\nu) \\ &+ \frac{4\mu\mathfrak{T}}{\omega_a L_a}\int_0^1 d\zeta\,|F(\mathfrak{a},\mathfrak{b},\mathfrak{c},\zeta)|^2\,[1-[2\zeta-1]\operatorname{arctanh}(2\zeta-1)]\,, \end{aligned} \tag{9}$$

with

$$\begin{aligned} \mathcal{R}_1(\nu) &= \int_{-\infty}^{\infty} d\omega\, i\omega^3 \frac{\hat{h}(\omega)-1}{\sinh^2\frac{\pi}{2}\sigma(\nu)\omega}, \\ \mathcal{R}_2(\nu) &= \int_{-\infty}^{\infty} d\omega\, \omega^2 \frac{\hat{h}(\omega)-1+\frac{1}{3}\omega\partial_\omega\hat{h}(\omega)}{\sinh^2\frac{\pi}{2}\sigma(\nu)\omega}. \end{aligned}$$

Here, $\hat{h}(\omega)$ is the Fourier transform of Raman response function from (3) with $h(\tau<0)\equiv 0$. Note that only the real parts of both integrands contribute to $\mathcal{R}_{1,2}(\nu)$. The dimensionless parameter μ is the input DW power P_b divided by the initial peak power of the soliton P_a

$$\mu = \frac{P_b}{P_a} \qquad \text{with} \qquad n_{2a}P_a = \frac{|\beta^{(2)}(\omega_a)|c}{\omega_a\sigma_a^2} = \frac{c}{\omega_a L_a},$$

where $L_a = \sigma_a^2/|\beta''(\omega_a)|$ is the dispersion length of the initial soliton. An adiabatic (slow) evolution of the soliton parameters requires $\mu \ll 1$.

Equations (7–9) provide self-consistent quantitative description of optical solitons controlled by collisions with DWs. They might look complicated, but are more easily accessible than the original full GNLSE. One can, e.g., easily quantify a DW that succeeds in cancellation of the SSFS, and the stability of the cancelation can be investigated, as we will see in the next section.

3 Controlled SSFS cancellation

To begin with, we switch off the DW and consider a single soliton ($\omega_a = 0.67$ PHz, $\sigma_a = 40$ fs) traveling along an optical fiber subject to Raman scattering, higher-order dispersion (Fig. 2), and

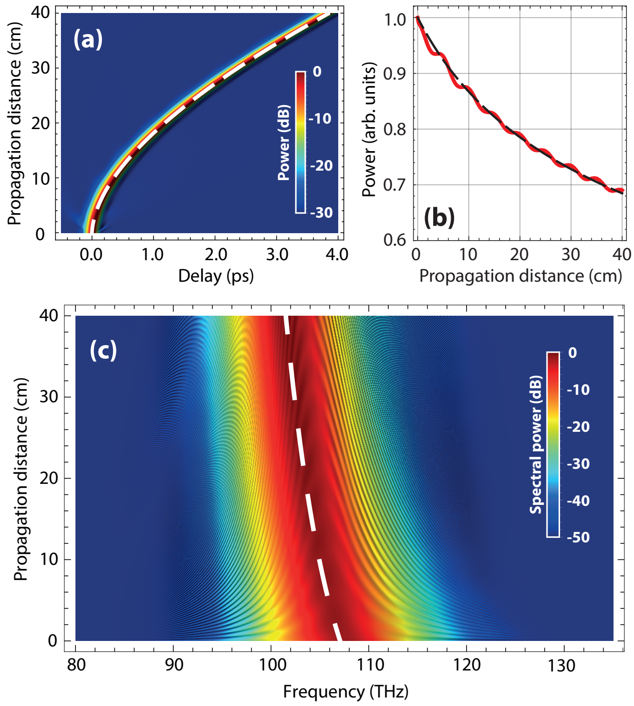

Figure 3: Space-time (a) and spectral (c) representation of a single soliton propagating along the fiber under influence of Raman effect from numerical simulation using full GNLSE. Additional dashed lines result from the adiabatic equations (7–9). (b) Soliton's peak power from the GNLSE (red) and from the adiabatic equations (dashed). See Sec. 3 for parameters.

self-steepening. We compare numerical solution of the full GNLSE (Fig. 3) to the predictions of the adiabatic equations (7–9) (dashed lines in Fig. 3). Note especially that the evolution of soliton power is accurately described, in contrast to predictions by standard soliton perturbation theory which state that the soliton amplitude is not affected by the Raman effect [1, 17].

Now we return to the scattering of the DW, as shown in Fig. 1(a) for the full GNLSE solution. Despite of the complex interaction character and approximations made when deriving (5), solutions of the above adiabatic equations (dashed lines in Fig. 1) provide reasonable quantitative estimates of the soliton trajectory and power, and an impressivly accurate prediction for the soliton carrier frequency. We can now derive initial parameter ranges such that a stable SSFS compensation arises.

For this we inspect the ODE (8) for soliton frequency shift, which is self-consistent due to (7). Its first summand describes the influence of Raman scattering, the second describes the influence of the imposed DW. We consider an exemplary initial soliton with $\omega_a = 0.67$ PHz and $\sigma_a = 40$ fs and look for pairs of DW frequency offset Ω and normalized intensity μ, such that soliton frequency evolves stable with $\nu(z) = \nu(0) = 0$ for all z. Fig. 4(a) shows DW parameters (Ω, μ) such that $\nu(z) \equiv 0$ is a stationary solution of (8), i.e., $(d\nu/dz)_{\nu=0}$ vanishes. The SSFS cancellation might be expected for $0.1 \leq \Omega/(\text{PHz}) \leq 0.3$, otherwise the required DW power quickly increases with μ and soliton evolution is no longer adiabatic. Furthermore, we can identify a parameter range for which the compensation is asymptotically stable. To this end

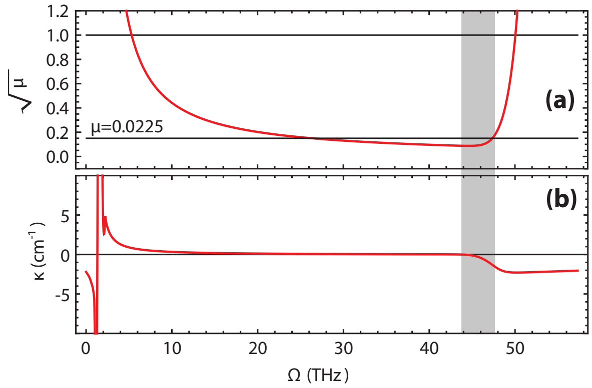

Figure 4: (a) Pairs of Ω and relative DW amplitude $\sqrt{\mu}$ for which solitons frequency does not change. Amplitude is plotted instead of power to facilitate readability. (b) Stability analysis. The region of stable SSFS compensation is shaded gray.

the derivative with respect to ν of the right-hand side of (8) should be negative for $\nu = 0$. The relevant quantity

$$\kappa = \partial_\nu \left(\frac{d\nu}{dz} \right) \bigg|_{\nu=0}$$

is shown in Fig. 4(b). For $\kappa > 0$ the SSFS reappears if fiber length exceeds $1/\kappa$. Within the small region of asymptotically stable ($\kappa < 0$) compensation, we must choose an initial parameter pair (Ω, μ) such that the relative DW power $\mu \ll 1$, in order for the adiabatic theory to be a good approximation. As threshold we generously choose $\mu < 0.0225$. The according parameter region is shaded gray in Fig. 4.

Fig. 5 shows the results of a simulation with parameters produced by the above procedure ($\Omega = 0.286$ PHz, $\mu = 0.0081$). The transient phase is very short. The soliton stabilizes at about 0.666 PHz (cf. with the initial $\omega_a = 0.67$ PHz). Fig. 5 (b) depicts soliton power evolution. We see that the soliton lost only 4% of its peak power after propagating a distance of 40 cm. Traveling alone, the soliton lost about 30% of its peak power due to the effects of Raman scattering, see Fig. 3(b).

In conclusion, we explained how the SSFS can be cancelled in an asymptotically stable manner by XPM interaction with a properly prepared small-amplitude wave, and provided a simple way to calculate the required wave parameters. U.B. and Sh.A. acknowledge support of Einstein Foundation Berlin and Research Center MATHEON under Project D-OT2.

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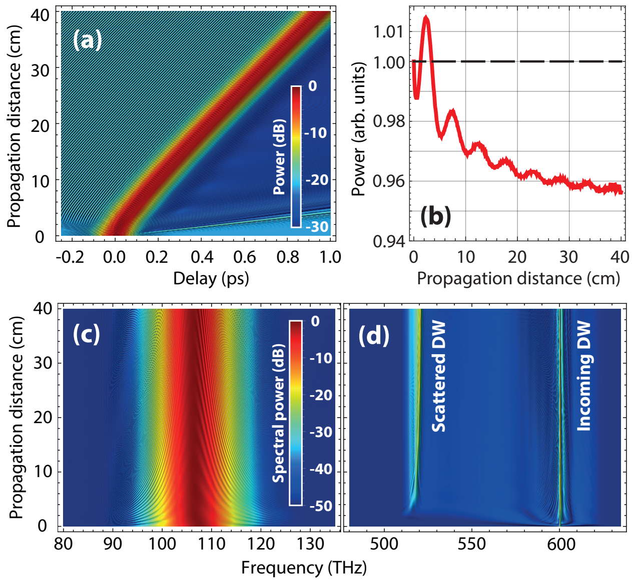

Figure 5: Compensation of SSFS by collision with a suitably prepared DW. (a) Evolution of soliton together with a DW in time domain as results from GNLSE. (b) Soliton amplitude is almost unchanged. (c) Soliton frequency is locked. (d) the scattered DW frequency is now well defined, cf., Fig 1(d). See Sec. 3 for parameters.

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