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Instability of shear waves in a nonuniform dusty plasma

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Abstract. It is shown that shear waves in a strongly coupled dusty plasma with nonuniform plasma pressure and dust density can be destabilized. The dispersion relation as well as the instability condition have been found and investigated in detail.

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1. Introduction

Since the first experimental creation of dust crystals [1]–[3], much effort has been devoted to understanding the main features of strongly coupled dusty plasmas (SCDPs) [4]–[10]. The correlation parameter for the α species of a plasma is $\Gamma_\alpha = Z_\alpha^2 e^2 / (a_\alpha T_\alpha)$, where Z_α is the charge number, e is the magnitude of the electron charge, $a_\alpha = (4\pi n_\alpha / 3)^{-1/3}$ is the mean spacing between the particles of the α component with density n_α and T_α is the temperature. While, in an ion–electron plasma, strong correlation ($\Gamma_i \gg 1$) can only be achieved in fusion devices or in stellar interiors, in a dusty plasma high coupling between dust grains ($\Gamma_d \gg 1$) and even crystallization of the dust sub-system ($\Gamma_d > \Gamma_c$) can easily be produced in an rf or dc plasma discharge, due to the large charge number, $Z_d \sim 10^4$, which dust grains may acquire [9]. In contrast to an ion–electron plasma, where electrons are degenerate for strong correlation of the ion sub-system [11, 12], in a dusty plasma dust species can be strongly correlated ($\Gamma_d \gg 1$) with other species that are weakly coupled and non-degenerate. In the present paper, attention is focused on the strongly coupled regime of the dust sub-system. In this limit, shear waves [10] were experimentally observed very recently for the first time after being theoretically predicted by Kaw and Sen [6] and numerically explored by Ohta and Hamaguchi [8]. It has been found that above a threshold pressure the dust acoustic longitudinal wave mode is dominant. On the other hand, when the pressure is reduced there arises a transverse mode which grows and takes over the acoustic-like wave. No explanation for the driving mechanism has been suggested so far. In the following, the collective behaviour of an SCDP liquid ($\Gamma_c \gg \Gamma \gg 1$) has been analysed and the resulting low-frequency modes are theoretically explored. A new source of instability has been suggested. The present theoretical approach, apart from exploring the combined influence of the plasma pressure and dust density gradients as sources for instability, differs from previous theoretical investigations in that it considers quasi-neutrality to hold for all orders of charge distribution, thus leading to a shear wave entirely supported by the mechanical rigidity of the medium. While the present theoretical and numerical analysis is helpful in understanding the main features of the dust-shear mode, the mechanism responsible for the instability of the experimentally observed transverse waves has not been identified.

The paper is organized as follows. In section 2 a theoretical model is presented. In section 3 the hydrodynamic and kinetic limits are analytically investigated. In section 4 thermodynamical quantities used in the theoretical approach are clarified in the light of previous theories [4] and numerical simulations [8]. In section 5 new experimental conditions are proposed and numerically explored. Our conclusions are contained in section 6.

2. Model equation

Using the general hydrodynamic (GH) model [11, 13, 14] approach, which extends the Navier–Stokes equation, one can write the dust equation of motion as

$$(1 + \tau_m \partial_t) \left[\rho_d (\partial_t + v_{dn} + \mathbf{v}_d \cdot \nabla) \mathbf{v}_d - Z_d e n_d \left(\mathbf{E} + \frac{1}{c} \mathbf{v}_d \times \mathbf{B} \right) + \nabla P_d + \rho_d \mathbf{g} \right] \\ = \eta \nabla \cdot \nabla \mathbf{v}_d + \left(\zeta + \frac{\eta}{3} \right) \nabla (\nabla \cdot \mathbf{v}_d), \quad (1)$$

where $\rho_d (=n_d m_d)$, \mathbf{v}_d and $Z_d e$ are the dust mass density, dust fluid velocity and dust charge, $P_d (=n_d T_d)$ is the dust pressure and η and ζ are the shear and bulk coefficients of viscosity [15], respectively. The electric and magnetic fields are denoted by \mathbf{E} and \mathbf{B} , respectively. The

viscoelastic relaxation time τ_m is an index of how memory effects will influence the shear wave propagation in the medium [14]. Using the quasi-neutrality condition $Z_d n_d + n_e = n_i$, and summing the equations of motion for inertialess ions and electrons, we obtain for the space charge electric field

$$Z_d e n_d \mathbf{E} = -\frac{\mathbf{j}_{e,i}}{c} \times \mathbf{B} - \nabla P_{e,i}, \quad (2)$$

which is valid in the low-frequency $\omega \ll kV_{Te}, kV_{Ti}$ and long-wavelength $\lambda \gg a$ limits, where ω is the wave frequency, k is the wavenumber and V_{Te} (V_{Ti}) is the electron (ion) thermal speed. Here, $\mathbf{j}_{e,i}$ ($=n_i e \mathbf{v}_i - n_e e \mathbf{v}_e$) and $P_{e,i}$ ($=n_i T_i + n_e T_e$) are the sums of the electron and ion currents and pressures. It should be noted that the quasi-neutrality condition used in equation (2) applies to all orders of the charge density, and not just to the zeroth order. Using equation (2) in (1) we obtain

$$(1 + \tau_m \partial_t) \left[\rho_d (\partial_t + v_{dn} + \mathbf{v}_d \cdot \nabla) \mathbf{v}_d + \frac{1}{c} \mathbf{j} \times \mathbf{B} + \nabla P + \rho_d \mathbf{g} \right] \\ = \eta \nabla \cdot \nabla \mathbf{v}_d + \left(\zeta + \frac{\eta}{3} \right) \nabla (\nabla \cdot \mathbf{v}_d), \quad (3)$$

where $\mathbf{j} = \mathbf{j}_{e,i} - n_d Z_d e \mathbf{v}_d$ and $P = P_{e,i} + P_d \approx P_{e,i}$ are the total plasma currents and plasma pressures, respectively. By using Faraday's law, namely $\nabla \times \mathbf{B} = 4\pi \mathbf{j}/c$, equation (3) can be written as

$$(1 + \tau_m \partial_t) \left[\rho_d (\partial_t + v_{dn} + \mathbf{v}_d \cdot \nabla) \mathbf{v}_d + \nabla \left(P + \frac{B^2}{8\pi} \right) - \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{4\pi} + \rho_d \mathbf{g} \right] \\ = \eta \nabla \cdot \nabla \mathbf{v}_d + \left(\zeta + \frac{\eta}{3} \right) \nabla (\nabla \cdot \mathbf{v}_d). \quad (4)$$

The equilibrium state, in the absence of the external magnetic field, reads

$$\rho_{d0} (v_{dn} \mathbf{v}_{d0} + \mathbf{g}) = -\nabla P_0, \quad (5)$$

where ρ_{d0} , \mathbf{v}_{d0} and P_0 are the equilibrium dust mass density, dust velocity and total plasma pressure, respectively. The linearization of equation (3) around the equilibrium in term of the perturbed dust number density ρ_{d1} , pressure P_1 and dust fluid velocity \mathbf{v}_{d1} can be written as

$$(1 + \tau_m \partial_t) \left[(\partial_t + v_{dn} + \mathbf{v}_{d0} \cdot \nabla) \mathbf{v}_{d1} + \frac{\nabla P_1}{\rho_{d0}} - \frac{\rho_{d1}}{\rho_{d0}^2} \nabla P_0 \right] \\ = \frac{\eta}{\rho_{d0}} \nabla \cdot \nabla \mathbf{v}_{d1} + \frac{1}{\rho_{d0}} \left(\zeta + \frac{\eta}{3} \right) \nabla (\nabla \cdot \mathbf{v}_{d1}). \quad (6)$$

Equation (6) reveals that the contribution of the plasma density fluctuations is contained in the perturbed pressure gradient ∇P_1 . The latter is related to the electrostatic potential, which in turn couples with the dust number density perturbation via the quasineutrality condition, as shown below.

We now take the curl of equation (6) and obtain

$$(1 + \tau_m \partial_t) \left[(\partial_t + v_{dn} + \mathbf{v}_{d0} \cdot \nabla) \nabla \times \mathbf{v}_{d1} - \frac{1}{\rho_{d0}^2} (\nabla \rho_{d0} \times \nabla P_1 + \nabla \rho_{d1} \times \nabla P_0) \right] \\ = \frac{\eta}{\rho_{d0}} \nabla \cdot \nabla (\nabla \times \mathbf{v}_{d1}) \quad (7)$$

which readily reduces to the case examined by previous authors [6] in the limit of a uniform collisionless medium without the equilibrium dust flow.

Writing $P_1 \approx n_{e1}T_e + n_{i1}T_i$ for $T_e \gg T_d$ and using the Boltzmann electron and ion density perturbations, namely $n_{e1} = n_{e0}e\phi/T_e$ and $n_{i1} = -n_{i0}e\phi/T_i$, we obtain $P_1 = -Z_d n_{d0}e\phi$, where we employed the quasi-neutrality condition, $n_{i0} = n_{e0} + Z_d n_{d0}$, for the unperturbed state. Here, T_e and T_i are the electron and ion temperatures, respectively, and ϕ is the electrostatic potential. On the other hand, from $n_{i1} = n_{e1} + Z_d n_{d1}$, we have

$$e\phi = -\frac{T_{e0}T_{i0}Z_d\rho_{d1}}{m_d(n_{e0}T_{i0} + n_{i0}T_{e0})}, \quad (8)$$

so that

$$P_1 = \frac{Z_d^2 n_{d0} T_e T_i}{m_d(n_{e0}T_i + n_{i0}T_e)}\rho_{d1} \equiv C_D^2 \rho_{d1}. \quad (9)$$

Here C_D represents the dust acoustic speed [16].

Let us consider the dust fluid velocity perturbation in the (x, y) plane only. In two dimensions, \mathbf{v}_{d1} may be expressed as a function of a scalar stream function $\varphi(x, z)$

$$\mathbf{v}_{d1} = \hat{\mathbf{y}} \times \nabla \varphi \quad (10)$$

where $\hat{\mathbf{y}}$ is the unit vector along the y axis, and $\Omega \equiv \nabla \times \mathbf{v}_{d1} = \hat{\mathbf{y}} \nabla_{\perp}^2 \varphi$ is the vorticity, which lies along the $\hat{\mathbf{y}}$ direction. The choice of \mathbf{v}_{d1} shows that the coupling between the dust flow and the equilibrium density gradient produces finite dust density perturbations. Linearizing the continuity equation for the dust fluid, we obtain

$$(\partial_t + \mathbf{v}_{d0} \cdot \nabla) \rho_{d1} = -\mathbf{v}_{d1} \cdot \nabla \rho_{d0}, \quad (11)$$

where we have noted that $\nabla \cdot \mathbf{v}_{d1} = 0$. The use of the latter in equation (1) consistently produces the quasineutrality condition $n_{i1} - n_{e1} - Z_d n_{d1} = 0$ if dust is cold.

Inserting equations (9) and (10) into (7), and making use of equation (11), we obtain

$$(1 + \tau_m \partial_t) \left[(\partial_t + \mathbf{v}_{dn} + \mathbf{v}_{d0} \cdot \nabla) (\partial_t + \mathbf{v}_{d0} \cdot \nabla) \nabla_{\perp}^2 \varphi - \frac{(\dot{P}_0 - C_D^2 \dot{\rho}_{d0}^2) \dot{\rho}_{d0}}{\rho_{d0}^2} \partial_x^2 \varphi \right] \\ = \frac{\eta}{\rho_{d0}} (\partial_t + \mathbf{v}_{d0} \cdot \nabla) \nabla^2 \nabla_{\perp}^2 \varphi, \quad (12)$$

where $\dot{P}_0 = dP_0/dz$ and $\dot{\rho}_{d0} = d\rho_{d0}/dz$. In equation (12), we have assumed that the equilibrium plasma pressure and dust density gradients are in the \hat{z} direction only.

We now Fourier transform equation (12) by letting $\varphi = \hat{\varphi} \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$, where \mathbf{k} is the wavevector. Using equation (5), we then obtain the desired dispersion relation

$$(1 - i\omega\tau_m) \left[(\omega - \mathbf{k} \cdot \mathbf{v}_{d0} + i\nu_{dn})(\omega - \mathbf{k} \cdot \mathbf{v}_{d0}) - \frac{(\nu_{d,n} \nu_{d0z} + g_0) \dot{\rho}_{d0}}{\rho_{d0}} \frac{k_x^2}{k_{\perp}^2} \right] \\ + i\eta^* (\omega - \mathbf{k} \cdot \mathbf{v}_{d0}) k^2 = 0, \quad (13)$$

where $g_0 = g + C_D^2 \dot{\rho}_{d0}/\rho_{d0}$ and $\eta^* = \eta/\rho_{d0}$. Equation (13) is valid in the local approximation which holds when the wavelength of the shear waves under consideration is much smaller than the scale sizes of the density gradient. In a fully ionized uniform plasma with $\omega \gg \mathbf{k} \cdot \mathbf{v}_{d0}, \nu_{dn}$, we have from equation (13)

$$\omega = -\frac{i\eta^* k^2}{(1 - i\omega\tau_m)} \quad (14)$$

which can be identified with the dispersion relation obtained in [7]. Equation (12) reveals the shear wave, $\omega = (\eta^*/\tau_m)^{1/2} k$, for $\omega \ll 1/\tau_m$.

3. Analytic results

3.1. Hydrodynamic limit ($\omega\tau_m \ll 1$)

As in the case of an ion–electron plasma [14], one can introduce a classification of the low-frequency modes in a SCDP based on the magnitude of $\omega\tau_m$. In the hydrodynamic limit, $\omega\tau_m \ll 1$, the dispersion relation (13) can be separated into an imaginary part

$$(2\gamma + \nu_{dn} + \eta^*k^2)(\omega_r - \mathbf{k} \cdot \mathbf{v}_{d0}) = 0, \quad (15)$$

and the real part

$$(\omega_r - \mathbf{k} \cdot \mathbf{v}_{d0})^2 - (\nu_{dn} + \gamma + \eta^*k^2)\gamma - \frac{\dot{\rho}_{d0}}{\rho_{d0}}(\nu_{d,n}\nu_{d0z} + g_0)\frac{k_x^2}{k_\perp^2} = 0, \quad (16)$$

where the notation $\omega \equiv \omega_r + i\gamma$ has been introduced. In this limit, the medium has a liquid-like behaviour and the viscosity, together with dust collisions with neutrals, is responsible for the wave damping. Since τ_m is the relaxation time of the dust medium, the frequency of the wave is much lower than the relaxation frequency in the hydrodynamic limit: the wave travels through the fluid-like medium where the components do not yet oscillate around fixed positions as in a crystal. On the other hand, in the kinetic limit ($\omega\tau_m \gg 1$), the wave motion is so rapid that the dust grains oscillate around a quasi-fixed position [4], as in a crystalline medium, acquiring the characteristic features of the modes propagating in a solid-like elastic medium. Defining $\beta = \nu_{dn} + \eta^*k^2$, we obtain for $\omega_r = \mathbf{k} \cdot \mathbf{v}_{d0}$ the imaginary part of the frequency

$$\gamma = -\frac{\beta}{2} \pm \frac{1}{2} \sqrt{\beta^2 - 4\frac{\dot{\rho}_{d0}}{\rho_{d0}}(\nu_{d,n}\nu_{d0z} + g_0)\frac{k_x^2}{k_\perp^2}}, \quad (17)$$

which has a positive solution when $(g_0 + \nu_{d,n}\nu_{d0z})\dot{\rho}_{d0} < 0$. In laboratory discharges on the ground, the dust density always decreases with distance from a negatively biased electrode leading always to at least one positive root. When $\dot{\rho}_{d0} = 0$, the dispersion relation yields a pure damped mode [6] with the imaginary frequency $\gamma = -(\nu_{dn} + \eta^*k^2)$.

For $\omega \neq \mathbf{k} \cdot \mathbf{v}_{d0}$ the real and imaginary parts are

$$\omega_r = \mathbf{k} \cdot \mathbf{v}_{d0} \pm \frac{1}{2} \sqrt{4\frac{k_x^2}{k_\perp^2}\frac{\dot{\rho}_{d0}}{\rho_{d0}}(\nu_{d,n}\nu_{d0z} + g_0) - (k^2\eta^* + \nu_{dn})^2} \quad (18)$$

and

$$\gamma = -\frac{\nu_{dn} + \eta^*k^2}{2}. \quad (19)$$

It is clear that, when the dust density gradient is negative or negligible, the latter limit always reduces to the previous one with an imaginary part as in (17). On the other hand, when the dust density gradient is positive and large, the real part can be modified giving rise to a mode which is highly damped by the neutral drag as well as by the viscoelastic behaviour of strongly correlated dust grains.

3.2. Kinetic limit ($\omega\tau_m \gg 1$)

With increasing ω , the features of the shear mode gradually shift to the so-called kinetic limit characterized by $\omega\tau_m \gg 1$: the wave motion is so fast that dust grains, on average, oscillate around equilibrium positions and the medium has an elastic-like behaviour. The properties of the wave change and the damping is no longer due to viscosity, as in the previous case, but to dust–neutral collisions. The wave is characterized by an elastic-type behaviour, in contrast to the previous viscous-like case. In the limit $\omega \gg \mathbf{k} \cdot \mathbf{v}_{d0}$, the real part of the dispersion relation is

$$\frac{(v_{dn}v_{d0z} + g_0)\dot{\rho}_{d0}}{\rho_{d0}} \frac{k_x^2}{k_\perp^2} + \frac{\eta^*k^2}{\tau_m} - \omega_r^2 + \gamma(\gamma + v_{dn}) = 0, \quad (20)$$

while the imaginary part is $\gamma = -v_{dn}/2$. The real part of the frequency can be written as

$$\omega_r = \pm \sqrt{\frac{(v_{dn}v_{d0z} + g_0)\dot{\rho}_{d0}}{\rho_{d0}} \frac{k_x^2}{k_\perp^2} + \frac{\eta^*k^2}{\tau_m} - \frac{v_{dn}^2}{4}}, \quad (21)$$

which reduces to

$$\omega_r = \pm k \sqrt{\frac{\eta^*}{\tau_m}}, \quad (22)$$

when $\omega \gg v_{dn}$ and $\dot{\rho}_{d0} = 0$. On the other hand, when $\omega_r = 0$ the dispersion relation depicts a purely imaginary mode with

$$\gamma = -\frac{v_{dn}}{2} \pm \frac{1}{2} \sqrt{v_{dn}^2 - 4 \left[\frac{\eta^*k^2}{\tau_m} + \frac{\dot{\rho}_{d0}}{\rho_{d0}} (v_{dn}v_{d0z} + g_0) \frac{k_x^2}{k_\perp^2} \right]}, \quad (23)$$

which is positive when

$$(v_{dn}v_{d0z} + g_0) \frac{\dot{\rho}_{d0}}{\rho_{d0}} < 0, \quad (24)$$

and $|\dot{\rho}_{d0}(v_{dn}v_{d0z} + g_0)| > (v_{dn}^2 - 4\eta^*k^2/\tau_m)\rho_{d0}k_\perp^2/k_x^2$. The kinetic limit has been investigated both with a GH approach and with the quasi-localized charge approximation QLCA model proposed in [4]. The QLCA model is based on the assumption that the wave motion is much faster than the dust grain diffusion time τ_d ; the average configuration of the system is described by a correlation function $g(r)$, where r is grain position. The dust grains can be randomly distributed as in disordered fluid but, if the wave is much faster than the diffusion time, the correlation function remains constant, and the dust grains move around fixed equilibrium positions. On the other hand, for wave frequencies much smaller than the diffusion frequency $\omega\tau_d \ll 1$, the average system configuration diffuses and the wave motion cannot be described by means of the QLCA model. In the literature, the viscoelastic relaxation time and the diffusion time are usually considered to be of the same order. There is though an intrinsic difference between the two characteristic times: the viscoelastic relaxation time is related to the mechanical property of the medium which acquires a crystalline-like structure increasing the correlation factor Γ . On the other hand, the diffusion time is related to the thermal relaxation of the average dusty plasma structure and varies with the dust thermal speed v_{thd} and the intergrain distance ($\tau_d \sim a/v_{thd}$), and it is not related to the mechanical rigidity of the dust medium. From those simple considerations one can argue that the QLCA can be successfully used to describe dust sound waves in a strongly coupled plasma where the dust sub-system has a solid-like random configuration, while the GH model is more suitable to explore dust sound and shear waves in dusty plasmas where the dust sub-system is highly packed and with a quasi-crystalline-like structure.

4. Thermodynamic quantities and limit of application

Before proceeding with detailed calculations, it is important to explore the range of validity of the theoretical model. The GH model considers the dust system as a fluid in such a way that the results are expected to be correct in the long-wavelength limit, i.e. when the normalized wavenumber $\tilde{k} = ka \ll 1$. The adiabatic constant γ_d had been taken to be 1.5 as suggested in [10]. The viscoelastic coefficient η has been normalized as $\tilde{\eta} = \eta/(\omega_{pd}\rho_{d0}a^2)$ and, following [12], has been taken to be $\tilde{\eta} = (1/30)\Gamma^{1/3}$. This relation holds when the coupling parameter $\Gamma > 10$ and away from the crystallization coupling value since, close to the Wigner crystallization Γ_c , the viscoelastic coefficient drastically increases due to the change in the mechanism of the momentum exchange between dust grains. The given expression for the viscoelastic coefficient η has been inferred from the OCP numerical simulations in the limit $\kappa = k_d a \rightarrow 0$, where k_d is the screening wavenumber. The memory coefficient τ_m is taken to be a function of $\tilde{\eta}$ as

$$\tilde{\tau}_m = \tau_m \omega_{pd} = \tilde{\eta} \frac{4 \omega_{pd}^2 a^2 m_d}{3 T_d} \frac{1}{1 - \gamma_d \mu_d + \frac{4}{15} \mu}. \quad (25)$$

Here μ_d is the compressibility which is a function of the coupling parameter and, in the OCP approximation, it can be expressed as $\mu_d = 1 + \mu(\Gamma)/3 + (\Gamma/9)\partial\mu(\Gamma)/\partial\Gamma$, where $\mu(\Gamma) = U_{ex}/T$ and U_{ex} is the excess energy. The total energy of a dusty plasma consists of kinetic and potential energies. The excess energy U_{ex} is the total potential energy in the limit $\kappa \rightarrow 0$. In the OCP approximation, the excess energy is equal to the correlation energy which can be calculated from simulations or statistical schemes. The OCP approximation [17] can be applied when the ratio of the intergrain distance a to the screening length λ_D is $\kappa = a/\lambda_D \ll 1$, i.e. when the grains are not shielded. On the other hand, when dust grains are shielded by the ion–electron cloud, the calculation of the excess energy is much more complex and represents the total Helmholtz free energy of the background plasma [8] which accounts for the free energy of each dust sheath. Previous numerical simulations have pointed out that, when the GH model with OCP excess energy description is applied to dusty plasmas with $\kappa \gg 1$, it gives results which are completely off the simulation data [8]. In the cited molecular dynamics simulations of a Yukawa system, since the plasma was homogeneous and the dust grains were initially in thermodynamic equilibrium, it was assumed that the screening was provided mainly by ions so that $\lambda_D \sim \lambda_{Di} = \sqrt{T_i/4\pi n_i e^2}$.

5. Numerical results

In this section, we suggest an experimental parameter regime where the given model, based on the GH theory, can be successfully used. The dust density is $n_d = 3.5 \times 10^4 \text{ cm}^{-3}$ and the grain radius is $R = 1.2 \text{ }\mu\text{m}$ while the grain mass has been chosen to be $m_d = 7.238 \times 10^{-12} \text{ g}$. The ion, neutral and dust temperatures are $T_i = T_n = T_d = 0.05 \text{ eV}$, while the electron temperature is $T_e = 0.4 \text{ eV}$. The levitation conditions have been found using the same numerical model as developed by the present authors to describe the dust grain dynamics in plasma sheaths, under low-pressure conditions, and successfully used in previous analyses [18]–[22]. It has been found that, in the empty sheath approximation, with an electrode potential $\phi = 3 \text{ V}$, the dust grains levitate at about 3.3 mm above the negative electrode and with a charge $Q = 5.1 \times 10^{-7} \text{ statcoulomb}$, corresponding to a charge number $Z_d = 1100$. The calculated grain charge has to be understood as a conservative maximum value, since it has been found neglecting the effect of the other grains in the sheath. On the other hand, a smaller Z_d would lead to a weaker coupling

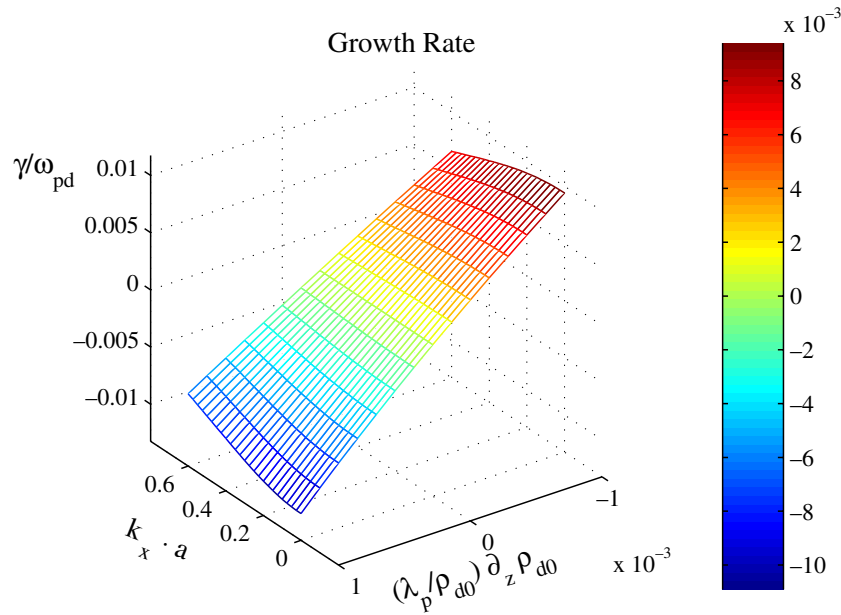


Figure 1. $T_i = T_n = T_d = 0.05$ eV, $T_e = 0.4$ eV, $n_d = 3.5 \times 10^4$ cm $^{-3}$, $n_i = 1.7 \times 10^7$ cm $^{-3}$, $n_e = 0.6 \times 10^7$ cm $^{-3}$, $m_d = 7.238 \times 10^{-12}$ g, $Z_d = 1100$, $P = 75$ mTorr, electrode potential $\phi = 3$ V, estimated levitation high above the negatively biased electrode ~ 3.3 mm and dust acoustic speed $C_D = 5.3$ cm s $^{-1}$.

that could be easily compensated by a dust density increase, i.e. by a reduction of the intergrain separation a . The ion density, in the plasma, is $n_i = 2 \times 10^7$ cm $^{-3}$ and decreases in the sheath, towards the electrode. For the given parameters, the ion density in the region where the grains oscillate is about $n_i \sim 1.7 \times 10^7$ cm $^{-3}$ with a Debye length $\lambda_{Di} = 416$ μ m, while the density of the thermalized electrons in the same region is $n_e = 0.6 \times 10^7$ cm $^{-3}$, resulting in an electron Debye length $\lambda_{De} = 2000$ μ m and in a dust acoustic speed $C_D \sim 5.3$ cm s $^{-1}$. The Wigner–Seitz radius $a = 190$ μ m is much shorter than both electron and ion Debye radii, and assuming that the screening is provided by the ions, the shielding parameter is $\kappa = 0.45 < 1$, which is consistent with the given limits of the GH model. The pressure has been chosen to be $P = 75$ mTorr and the resulting normalized dust–neutral frequency is $\tilde{\nu}_{dn} = 0.1387$.

5.1. Hydrodynamic limit ($\omega\tau_m \ll 1$)

From the suggested parameters the Coulomb coupling parameter is $\Gamma \sim 183$, while the nonlocal viscoelastic operator, which accounts for memory effects, can be estimated in normal units to be $\tilde{\tau}_m \sim 2.172$. Since the dust grains are shielded, the average interparticle potential energy decreases exponentially with the parameter $\kappa = a/\lambda_D$ and the effective coupling parameter is $\Gamma_d = \Gamma \exp(-\kappa)$. On the other hand, in the selected operational range, the shielding is extremely weak ($\kappa \ll 1$) because of the high packing, and the large Debye radius. When $v_{d0} = 0$, the real part of the frequency $\omega_r = \mathbf{k} \cdot \mathbf{v}_{d0} = 0$, while the growth rate is expressed by equation (17).

For small wavenumber k , the dust–neutral collision damping is dominant, since the viscosity is $\tilde{\eta} = 0.1895$ while the dust–neutral collision frequency is $\tilde{\nu}_{dn} = 0.1387$. In figure 1 the imaginary part of the frequency is shown as a function of the normalized wavenumber $\tilde{k}_x = k_x a$

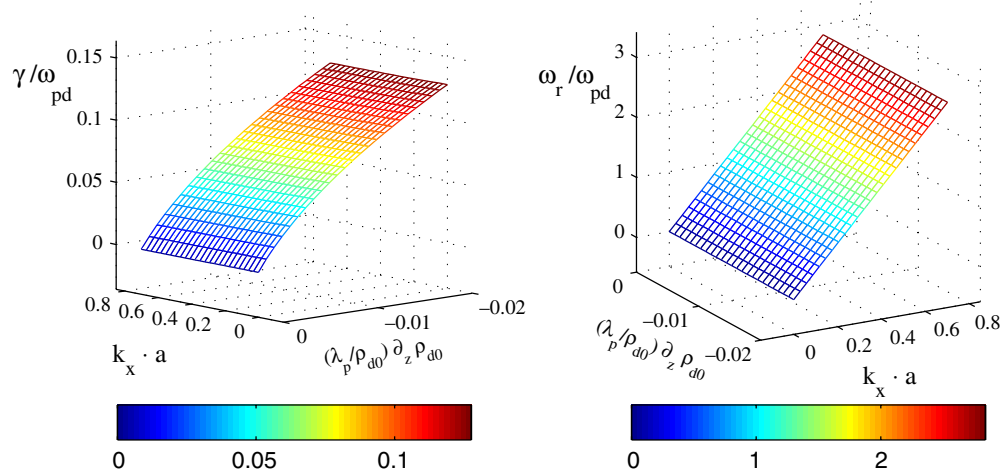


Figure 2. Parameters: as in figure 1 except for $v_{d0} \sim 10 \text{ cm s}^{-1}$.

and of the destabilizing parameter $\lambda_{Di}/(\rho_{d0}\partial_z\rho_{d0})$, with no initial drift. Figure 1 shows that a negative dust density gradient leads to a wave growth which decreases slightly with \tilde{k}_x . The destabilizing effect of the negative dust density gradient $\partial_z\rho_{d0}$ is not negligible even for a conservative value as low as $\rho_{d0}/10^3\lambda_{Di}$. Larger values of the dust density gradient lead to much larger growth rates, a characteristic of the kinetic regime which will be examined in the next section. A first evaluation of the destabilizing factor, $\dot{\rho}_{d0}$, which leads to instability, suggests that it must be negative and of the order of $\dot{\rho}_{d0}/\rho_{d0} \sim v_{dn}^2/g$. A linear approximation of the dust density gradient $\partial_z\rho_{d0}/\rho_{d0} = 1/(\lambda_{Di}L)$ suggests that even a conservative $L \sim 1 \times 10^3$ gives a considerable imaginary response. The role of the wavenumber k is to decrease the importance of the dust density gradient, either when driving or damping the wave. In figure 1 the characteristic gradient length L is chosen to be within the range $(0.5 \times 10^3 - 10^3)$. For the present choice of the parameters, \mathbf{k} is along the x direction, implying that $\mathbf{k}_\perp = \mathbf{k} = \hat{x}k_x$.

5.2. Kinetic limit ($\omega\tau_m \gg 1$)

Using the given parameters, the nonlocal viscoelastic operator $\tilde{\tau}_m$, which accounts for the memory effect, is of the order of unity ($\tilde{\tau}_m = 2.178$). To reach the kinetic limit regime the wave frequency has to be $f > \omega_{pd}/2\pi \sim 20 \text{ Hz}$ or, in other words, the normalized wave growth rate needs to be significantly larger than unity. The kinetic shear wave regime has been previously analysed in the limit $\omega \gg \mathbf{k} \cdot \mathbf{v}_{d0}$, and in the opposite case $\omega = 0 \ll \mathbf{k} \cdot \mathbf{v}_{d0}$ since the theoretical expression for the growth rate when $\omega \sim \mathbf{k} \cdot \mathbf{v}_{d0}$ was too complex. From equation (23) we can obtain that the dust drift velocity, in order to influence the growth rate, has to be about $g/v_{dn} \sim 55 \text{ cm s}^{-1}$. In figure 2 the real and imaginary parts of the frequency are represented as a function of the normalized wavenumber $\tilde{k} = ka$ and of the driving parameter $(\lambda_{Di}/\rho_{d0})\partial_z\rho_{d0}$, where the previously defined gradient characteristic factor L has been chosen to be in the conservative range $[0, 50]$ and the drift speed is $v_{d0} = 10 \text{ cm s}^{-1}$. From figure 2, it can be seen that the real part of the frequency $\omega_r = \mathbf{k} \cdot \mathbf{v}_{d0}$. The growth rate is strongly dependent on the dust density gradient, while its variation with k is almost negligible. In the present calculations, \mathbf{k} is along the x axis as already mentioned, resulting in a k^2 dependence of the mode growth rate γ .

6. Conclusions

In this paper, we have investigated instability of low-frequency shear waves in an SCDP. The GH formalism has been applied and new driving sources, namely the combined influence of the plasma pressure and dust density gradients, have been theoretically analysed. Our results show that shear waves can become unstable in a nonuniform dusty plasma containing plasma pressure and dust density gradients. Physically, instability is caused by gravity induced acceleration of strongly coupled dust grains in a nonuniform plasma [23].

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