# Supplementary Materials for: Chiral surface twists and skyrmion stability in nanolayers of cubic helimagnets

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## I. Solutions for confined modulations in cubic helimagnet nanolayers

Magnetic states in bulk cubic helimagnets are commonly described by *unconfined* modulated states and derived by minimization of the energy functional (Eq. 1)[1]

$$w = A\,(\mathbf{grad}\,\mathbf{m})^2 + D\,\mathbf{m}\cdot\mathrm{rot}\,\mathbf{m} - \mu_0\,M\mathbf{m}\cdot\mathbf{H}, \tag{1}$$

in an infinite system[2,4,5]:

**(i)** *Cones* are chiral single-harmonic modulations along the applied field. The solutions for the cone phase and the equilibrium energy density are derived in analytical form[5]

$$\cos\theta_c = h, \quad \psi_c = 2\pi z/L_D, \quad w_c(h) = -K_0\left(1+h^2\right) \tag{2}$$

where $K_0 = D^2/(4A) = \mu_0 H_D M/2$ is the effective easy-plane anisotropy imposed by the cone modulations[6,7].

**(ii)** *Helicoids* are one-dimensional chiral modulations with the propagation direction perpendicular to the applied field and homogeneous along the direction of the applied field[2]. Helicoids propagating along the $x$-axis are described by solutions $(\theta(x), \psi = \pi/2)$. The Euler equation for the helicoid energy density

$$w_h^0(\theta) = A\,\theta_x^2 - D\,\theta_x - \mu_0 MH\cos\theta \tag{3}$$

yields a set of parametrized periodic solutions $\theta(x,l)$ where the parameter $l$ designates the period of helicoids. The equilibrium period $l_0$ and profile $\theta(x,l_0)$ are derived by minimization of the helicoid energy density with respect to $l$[2].

**(iii)** *Skyrmion lattices.* The axisymmetric cores of chiral skyrmion lattice cells are described by solutions[4]

$$\theta(\rho), \quad \psi = \pi/2 + \varphi \tag{4}$$

where $\mathbf{r} = (\rho\cos\varphi, \rho\sin\varphi, z)$ are cylindrical coordinates of the spatial variable.

The equilibrium periods and magnetization profiles $\theta(\rho)$ of the skyrmion lattice cells are derived by minimization of the energy density functional[4]

$$w_s^0(\theta) = A\,\mathcal{J}_s^0(\theta) + D\,\mathcal{I}_s^0(\theta) - \mu_0 MH\cos\theta, \tag{5}$$

$\mathcal{J}_s^0(\theta) = \theta_\rho^2 + \frac{1}{\rho^2}\sin^2\theta$, $\mathcal{I}_s^0(\theta) = \theta_\rho + \frac{1}{\rho}\sin\theta\cos\theta$ for different values of the core radii $R$, and optimization of the mean energy density of the skyrmion lattice with respect to $R$.

Among these solutions, the cone phase (2) corresponds to the global minimum of model (1) over the whole region where chiral modulations occur ($H < H_D$). The helicoids and skyrmion lattices exist as metastable states below the critical fields $H_h = 0.617 H_D$ and $H_s = 0.801 H_D$ correspondingly[2,4].

The energy density functional for confined helicoids ($w_h(\theta, \psi)$) and skyrmion lattices ($w_s(\theta, \psi)$) can be written in the following form

$$w_{h(s)} = A\mathcal{J}_{h(s)}(\theta, \psi) + D\mathcal{I}_{h(s)}(\theta, \psi) - \mu_0 M H \cos\theta, \quad (6)$$

where the exchange ($\mathcal{J}_{h(s)}$) and Dzyaloshinskii-Moriya ($\mathcal{I}_{h(s)}$) energy functionals read as

$\mathcal{J}_h(\theta, \psi) = \theta_x^2 + \theta_z^2 + \sin^2\theta\,(\psi_x^2 + \psi_z^2)$,

$\mathcal{I}_h(\theta, \psi) = \cos\psi\theta_x + \sin\theta\cos\theta\sin\psi\ \psi_x + \sin^2\theta\,\psi_z$,

$\mathcal{J}_s(\theta, \psi) = \theta_\rho^2 + \theta_z^2 + \sin^2\theta\left(\frac{1}{\rho^2}\psi_\varphi^2 + \psi_z^2\right)$,

$\mathcal{I}_s(\theta, \psi) = \sin(\psi - \varphi)(\theta_\rho + \frac{1}{\rho}\sin\theta\cos\theta\,\psi_\varphi) + \sin^2\theta\,\psi_z$.

The equilibrium solutions for confined helicoids and skyrmion lattices are derived by solving the Euler equations for functional (6) with free boundary conditions at the film surfaces ($z = \pm L/2$).

### II. Analytical solutions for surface twists

In cubic helimagnet films with $L \geq L_D$, twisted modulations in helicoids and skyrmions ($\xi(z)$) exist only in narrow regions near the film surfaces $\delta \ll L$. This allows us to write solutions for helicoids as $\theta = \theta(x), \psi = \pi/2 + \xi(z)$ (the $x$ axis is directed along the propagation direction), where $\theta(x)$ is the solution homogeneous along the $z$ axis investigated in[2]. We write the solutions for a skyrmion lattice core as $\theta = \theta(\rho), \psi = \pi/2 + \varphi + \xi(z)$ where and $\theta(\rho)$ is the solutions for skyrmions homogeneous along their axes[4]. The energy density of the surface twists in the helicoid (skyrmion lattice) can be reduced to the following form: $e_{h(s)}(\xi) = \Delta\bar{w}_{h(s)}(\xi) = \langle m_x^2\rangle_{h(s)}\,\mathcal{F}_{h(s)}(\xi)$ where

$$\mathcal{F}_{h(s)}(\xi) = \frac{2}{L}\int_0^\infty dz\left[A\xi_z^2 - D\xi_z - K_0\,\upsilon_{h(s)}\sin^2\frac{\xi}{2}\right], \quad (7)$$

where $K_0$ is the effective anisotropy (2), and

$$\left\langle m_x^2 \right\rangle_h = \frac{1}{l}\int_0^l \sin^2\theta dx,\ \left\langle m_x^2 \right\rangle_s = \frac{1}{\pi R_s^2}\int_0^{R_s} \sin^2\theta\rho d\rho,\ \upsilon_h = (4L_D/l)\left\langle m_x^2 \right\rangle_h^{-1},$$
$$\upsilon_s = 2\eta_D(L_D/\pi)\left\langle m_x^2 \right\rangle_s^{-1},\ \eta_D = \frac{1}{\pi R_s^2}\int_0^{R_s}\left(\theta_\rho + \frac{1}{\rho}\sin\theta\cos\theta\right)\rho d\rho \tag{8}$$

The energy functional $\mathcal{F}_{h(s)}(\xi)$ (7) describes surface twists $\xi(z)$ in helicoids (skyrmion lattices) and has the same functional form as the energy functional for surface twists in a saturated helimagnet[8]. The Euler equation for (7) can be readily solved analytically. The equilibrium amplitude of twist modulations $\xi(z)$ reaches the largest value on the film surface,

$$\xi_{h(s)}^{(0)} = 2\arcsin\left(\upsilon_{h(s)}^{-1/2}\right) \tag{9}$$

and decays exponentially into the layer depth,

$$\tan(\xi_{h(s)}/4) = \tan(\xi_{h(s)}^{(0)}/4)\, e^{\left[-\pi\sqrt{\upsilon_{h(s)}}(z/L_D)\right]}. \tag{10}$$

Inserting (10) into the energy density (7) leads to the following expression for the *negative* energy density contribution imposed by surface twist modulations:

$$\bar{e}_{h(s)} = \underbrace{\left\langle m_x^2 \right\rangle_{h(s)}\left[2\tan\left(\xi_{h(s)}^{(0)}/4\right) - \xi_{h(s)}^{(0)}\right]}_{\sigma_{h(s)}}/L < 0. \tag{11}$$

The fractions of the *negative* surface contribution $\bar{e}_{h(s)} \propto 1/L$ (11) in the total energy balance increase with decreasing film thickness, extending the stability areas of the helicoids and skyrmion lattices (Fig. 2 in[1]).

### III. Thin film preparation of FeGe helimagnet

A wedge-shape (110) film for TEM observations is made from FeGe single crystal by using a focused ion beam technique. A typical Lorentz image is given in Fig. S2[9,10]. The film thickness profile was examined by using EELS spectra as shown in the inset of Fig. S2(a), presenting the wedge-shape profile of the film with the thickness of the edge starting from 40 nm. The field views in Figs. 3 and 4 in the main text are given in Fig. S2(b), respectively.

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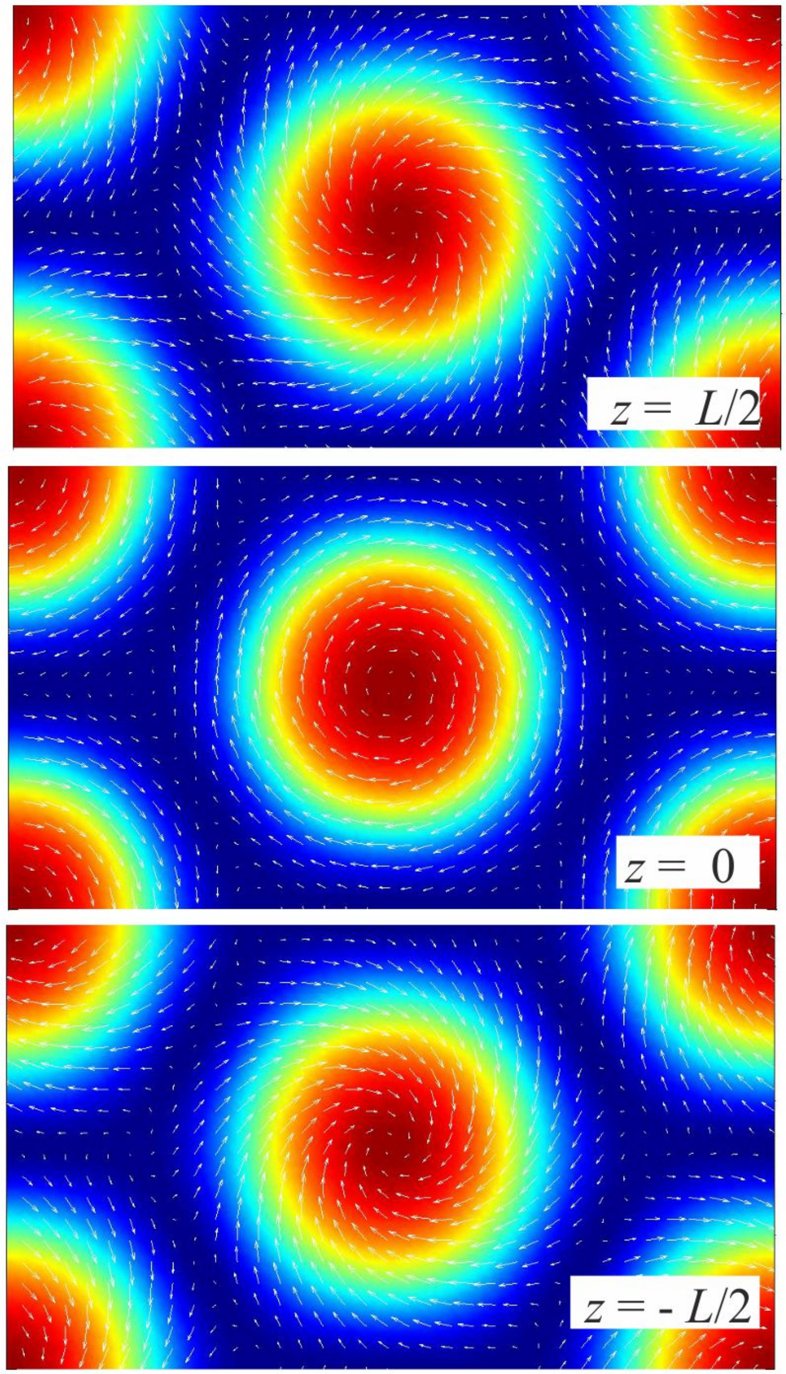


FIG. 1. (color online). Contour plots of the magnetization $m_z(x, y)$ in the skyrmion lattice cell for top ($z = L/2$), middle ($z = 0$), and bottom ($z = -L/2$) of the film (Fig. 1 of the main text[1]) indicate longitudinal modulations through the film thicknes (*chiral twists*).

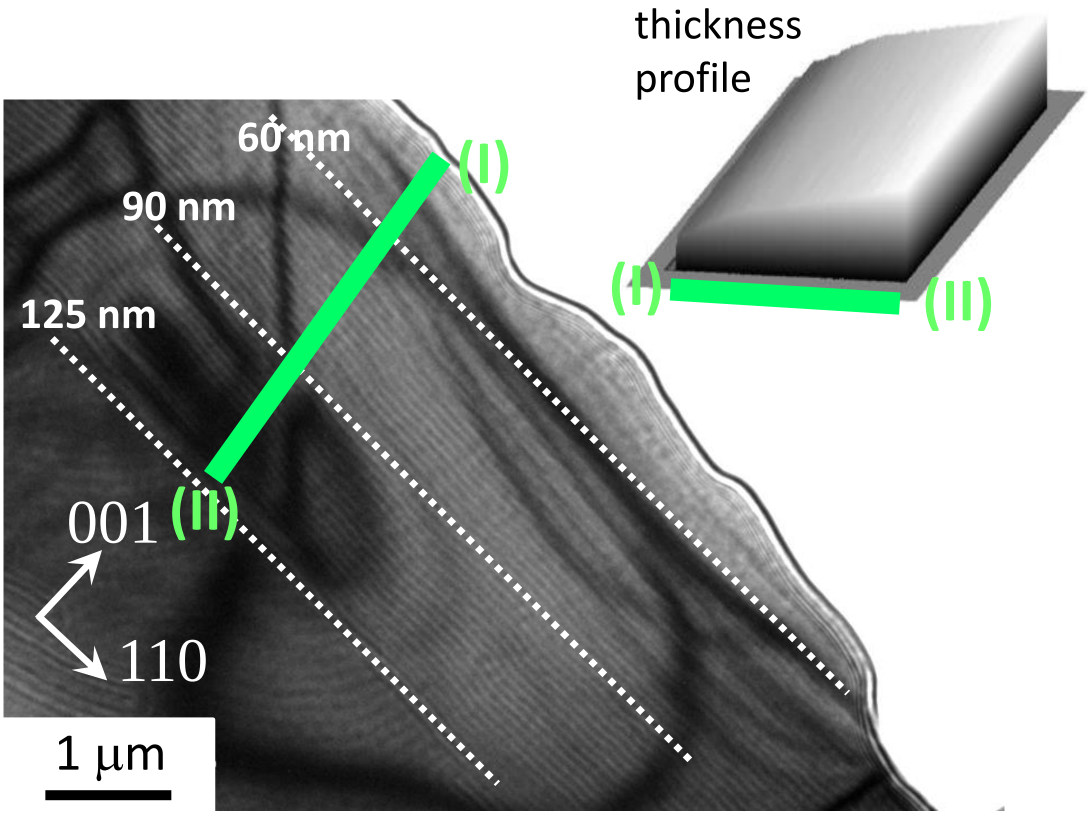


FIG. 2. (color online). A wedge-shape FeGe (110) film for TEM observations.

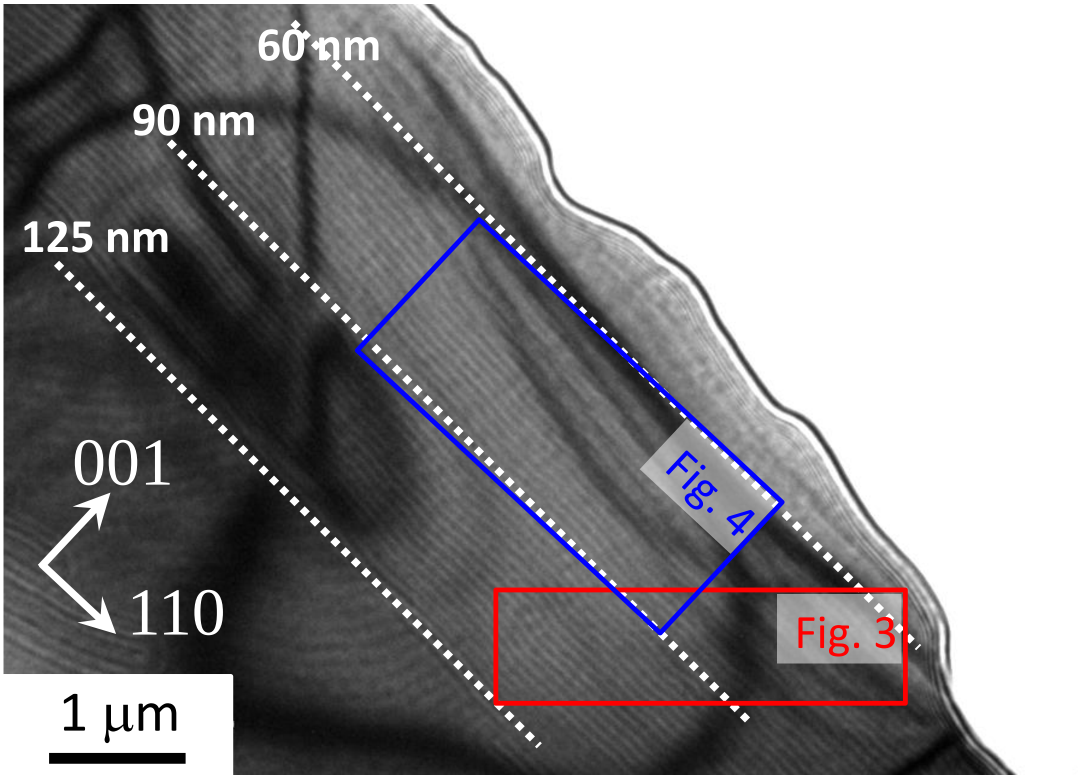


FIG. 3. (color online). A typical Lorenz image in a wedge-shape FeGe (110) film. The rectangles indicate the areas of the Lorenz images presented in Fig. 3 and Fig. 4 of the main text[1].