Original Paper

Investigation into glass tank geometries by means of a mathematical model

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On the basis of fundamental studies of the mechanisms of the glass melting process, a mathematical model of the flow and heat transfer in glass tank furnaces is proposed. The results obtained from the mathematical model agree well with those obtained from an earlier developed physical model.

Investigations of different glass tank geometries were performed by means of this mathematical model. The temperature and flow velocity profiles of the glass melt as well as the back-flow coefficient for various types of glass tank geometries were calculated based on the mathematical model. The optimum location of a weir in a container glass tank was studied. The decrease of the depth of the conditioning zone and the installation of a weir at the entrance of the neck have also been considered for a float glass tank. From the analysis of the mathematical model, an improved energy-saving glass furnace construction is put forward. The results of the described mathematical model are expected to be valuable in the design as well as in the research and development of glass tank furnaces.

Untersuchung der Geometrie von Glasschmelzwannen mit Hilfe eines mathematischen Modells

Es wird ein mathematisches Modell für den Strömungs- und Wärmetransport in Glasschmelzwannenöfen vorgeschlagen, das auf grundlegenden Überlegungen zum Mechanismus des Glasschmelzprozesses beruht. Die mit diesem Modell erhaltenen Ergebnisse stimmen gut mit denen eines früher entwickelten physikalischen Modells überein.

Untersuchungen des Einflusses der verschiedenen Wannengeometrien auf den Glasschmelzprozeß werden mit Hilfe des mathematischen Modells durchgeführt. Hierzu werden die Temperatur- und Strömungsgeschwindigkeitsprofile sowie der Rückströmungskoeffizient für die unterschiedlichen Ofengeometrien auf der Basis des mathematischen Modells berechnet. Die optimale Anordnung eines Walls in einer Behälterglaswanne wird untersucht. Die Verminderung der Tiefe der Abstehwanne und der Einbau eines Walls im Bereich der Einschnürung werden auch für eine Floatglaswanne betrachtet. An Hand der mathematischen Modellanalyse wird eine verbesserte energiesparende Ofenkonstruktion vorgestellt. Es wird damit gerechnet, daß die Ergebnisse des mathematischen Modells sowohl für die Ofenauslegung als auch für Forschung und Entwicklung im Bereich des Glasschmelzwannenbaus von Bedeutung sind.

1. Introduction

Controlling the glass currents and flow patterns plays an important role in improving the tank furnace operation and increasing the output and quality of glasses as well as in energy savings. Therefore, a good deal of attention has been paid to this for many years by engineers and operators. It is difficult, however, to do research in this field owing to the complex processes of physical chemistry and mass transfer in glass tanks. In general, there are three methods for research to make use of: measurement on full-scale glass tanks and physical as well as mathematical modeling [1 to 6]. In this paper a two-dimensional mathematical model will be described.

2. Mathematical model

The main assumptions for the mathematical model are: The molten glass is a homogeneous, incompressible Newtonian viscous fluid, neglecting bubbles and chemical reactions, the flow and heat transfer of the glass in tank furnaces are steady-state.

Some physical properties of glass, such as the specific heat capacity and the coefficient of thermal expansion, apparently do not vary with temperature and can be approximately considered as constants. Whereas the viscosity, μ, thermal conductivity, Γ_{eff}, and density, ϱ, intensely vary with temperature and affect glass currents and heat transfer significantly. The latter are expressed as functions of temperature in the following formulas:

$$\mu = 10^{-A_c + B_c/(\vartheta - \vartheta_c)}\,, \qquad (1)$$

$$\Gamma_{\text{eff}} = k_0 + k_1\,\vartheta + k_2\,\vartheta^2 + k_3\,\vartheta^3, \qquad (2)$$

$$\varrho = \varrho_0\,[1 - \beta\,(\vartheta - \vartheta_{\min})] \qquad (3)$$

where A_c, B_c, ϑ_c, k_0 to k_3 are constants, β is the coefficient of cubical expansion, C is the specific heat capacity and ϱ_0 is the density at temperature $\vartheta_{\min}$.

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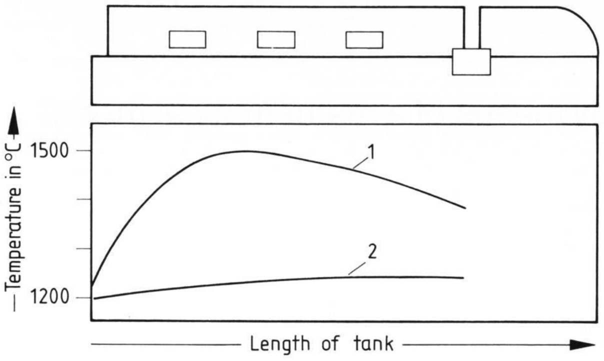


Figure 1. Principle sketch of a container glass tank (pull: 70 t glass/d) and temperature distribution at the bath surface (curve 1) and at the tank bottom (curve 2).

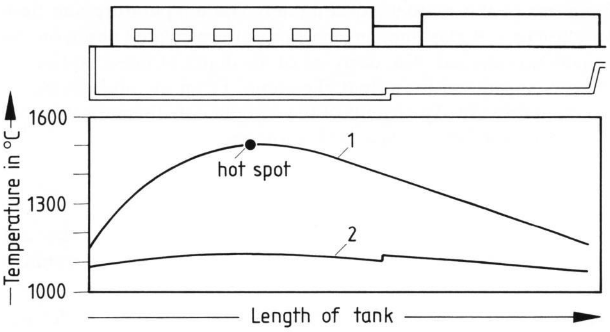


Figure 2. Principle sketch of a float glass tank (pull: 250 t glass/d) and temperature distribution at the bath surface (curve 1) and at the tank bottom (curve 2).

A two-dimensional mathematical model of the flow and heat transfer in glass tank furnaces can be described by a group of the partial differential equations, involving the continuity (4), momentum (5 and 6), and energy equations (7):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \tag{4}$$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} 2\,\mu \frac{\partial u}{\partial x} + \frac{\partial}{\partial y} \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right); \tag{5}$$

$$0 = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial y} 2\,\mu \frac{\partial v}{\partial y} + \frac{\partial}{\partial x} \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + g\,\varrho; \tag{6}$$

$$u \frac{\partial \vartheta}{\partial x} + v \frac{\partial \vartheta}{\partial y} = \frac{\partial}{\partial x} a \frac{\partial \vartheta}{\partial x} + \frac{\partial}{\partial y} a \frac{\partial \vartheta}{\partial y} \tag{7}$$

where x and y are the coordinates in longitudinal and in depth direction, u and v are the velocity components in x and y directions, respectively, ϑ is the temperature, p is the pressure and μ is the viscosity of molten glass, $a = K/C$ is the thermal diffusivity of glass. The inertia terms in the momentum equations are neglected because of the small Reynolds number.

The stream function and vorticity are usually introduced by

$$u = \frac{\partial \psi}{\partial y}, \tag{8}$$

$$v = \frac{\partial \psi}{\partial x}, \tag{9}$$

$$\omega = -\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right). \tag{10}$$

For calculating conveniently, equations (4 to 10) are expressed in dimensionless form. The dimensionless variables are:

$X = x/h$, $\quad Y = y/h$,
$\eta = \mu/\mu_0 = \mu(\vartheta)/\mu(\vartheta_{min})$, $\quad \Gamma = \lambda_{eff}/\lambda_{eff}(\vartheta_{min}) = \lambda_{eff}(\vartheta)/\lambda_{eff}(\vartheta_{min})$,
$A = a/a_0 = a(\vartheta)/a(\vartheta_{min})$, $\quad \Psi = \psi/a_0$,
$\Omega = \omega\, h^2/a_0$, $\quad \theta = \dfrac{\vartheta - \vartheta_{min}}{\vartheta_{max} - \vartheta_{min}}$.

Thus, equations (4 to 7) are changed to the following expressions:

$$\eta \cdot \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2}\right) + 2\frac{d\eta}{d\theta}\left(\frac{\partial \theta}{\partial X}\frac{\partial \Omega}{\partial X} + \frac{\partial \theta}{\partial Y}\frac{\partial \Omega}{\partial Y}\right) = 4\left(\frac{d\eta}{d\theta}\frac{\partial^2 \theta}{\partial X \partial Y} + \frac{d^2\theta}{d\theta^2}\frac{\partial \theta}{\partial X}\frac{\partial \theta}{\partial Y}\right)\frac{\partial^2 \Psi}{\partial X \partial Y} - Ra\frac{\partial \theta}{\partial X} +$$

$$+ \left[\frac{d\eta}{d\theta}\left(\frac{\partial \theta}{\partial Y^2} - \frac{\partial^2 \theta}{\partial X^2}\right) + \frac{d^2\eta}{d\theta^2}\left(\frac{\partial \theta}{\partial Y} + \frac{\partial \theta}{\partial X}\right)\left(\frac{\partial \theta}{\partial Y} - \frac{\partial \theta}{\partial X}\right)\right]\left(\frac{\partial^2 \Psi}{\partial Y^2} - \frac{\partial^2 \Psi}{\partial X^2}\right), \tag{11}$$

$$\Omega = -\left(\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2}\right), \tag{12}$$

$$0 = A\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) + \left(\frac{\partial A}{\partial X} - \frac{\partial \Psi}{\partial Y}\right)\frac{\partial \theta}{\partial X} +$$

$$+ \left(\frac{\partial A}{\partial Y} + \frac{\partial \Psi}{\partial X}\right)\frac{\partial \theta}{\partial Y} \tag{13}$$

where $Ra = g\,\varrho_0\,\beta\,(\vartheta_{max} - \vartheta_{min})\; h^3/(a_0\,\mu_0)$ (Ra = Rayleigh number). ϑ_{max} is the maximum temperature and ϑ_{min} is the minimum temperature, h is the depth of molten liquid in the tank.

3. Boundary conditions

Using this mathematical model a container glass tank (pull: 70 t glass/d) and a float glass tank (pull: 250 t glass/d) were calculated with regard to the temperature and velocity profiles. The temperature boundary conditions can be seen in figures 1 and 2.

The calculation of the melting velocity of the batch yields a parabolic distribution:

$$u(x) = \frac{3\,F_B}{2\,\varrho_B\,L_B\,W}\left[1-4\left(\frac{X-X_c}{L_B}\right)^2\right] \tag{14}$$

where F_B is the rate of batch feeding, ϱ_B is the density of batch, L_B is the length of batch pile and W is the width of the tank, $X_c = L_B/2$.

The pull velocity in the chute or in the throat results also in a parabolic distribution:

$$u(y) = M\,\frac{F_B}{\varrho_B\,W\,h_c}\left[1-4\left(\frac{Y-Y_c}{h_c}\right)^2\right] \tag{15}$$

where h_c is the exit height of the float glass tank or the throat height of the container glass tank, $Y_c = h_c\,I_c$. At the bottom and the side wall of the glass tank

$$u = v = 0 \tag{16}$$

holds. Calculated with equation (15 and 16) $I_c = 1/2$ is obtained for the container glass tank and $I_c = 1.1$ for the float glass tank if M = constant. Additionally at the free surface

$$\frac{\partial u}{\partial y} = v = 0 \tag{17}$$

holds and at the interface between batch and glass melt on condition that

$$u = 0\,, \tag{18}$$

$$v = -\,v_B(x) \tag{19}$$

equations (20 and 21) are obtained:

$$\psi = \frac{3\,F_B}{2\,\varrho_B\,W}\left[\frac{1}{3} + \frac{X-X_c}{L_B} - \frac{4}{3}\left(\frac{X-X_c}{L_B}\right)^3\right], \tag{20}$$

$$\omega = \frac{12\,F_B}{\varrho_B\,W\,L_B^2}\left(\frac{X-X_c}{L_B}\right). \tag{21}$$

For

$$u = u(y), \tag{22}$$

$$v = 0 \tag{23}$$

at the outlet yield:

$$\psi = M\,\frac{F_B}{\varrho_B\,W}\left[\frac{1}{3} + \frac{Y-Y_c}{h_c} - \frac{4}{3}\left(\frac{Y-Y_c}{h_c}\right)^3\right], \tag{24}$$

$$\omega = 8\,M\,\frac{F_B}{\varrho_B\,W\,h_c^2}\left(\frac{Y-Y_c}{h_c}\right). \tag{25}$$

4. Numerical solution

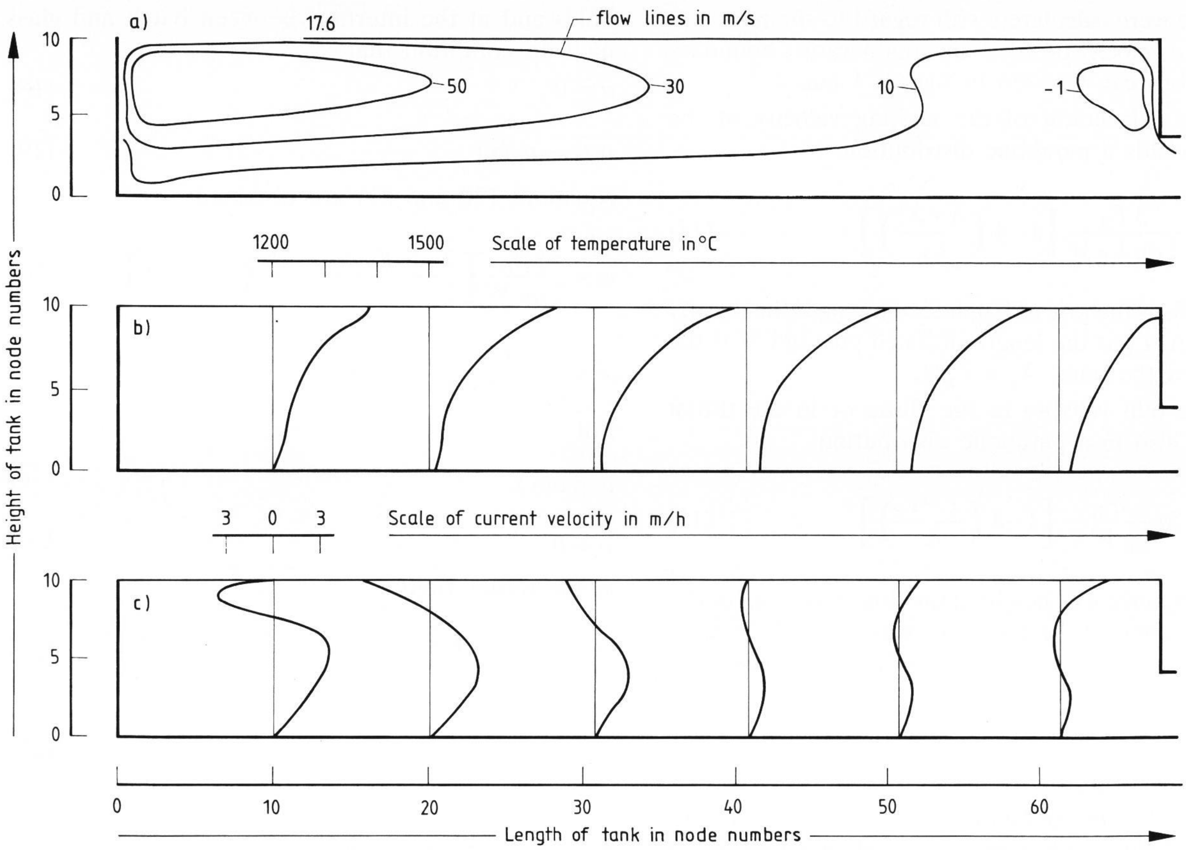
The finite difference method is applied to the solution of equations (11 to 13). The two-dimensional difference grid of the container glass tank is (68 × 10), and

Table 1. Physical properties of investigated glasses

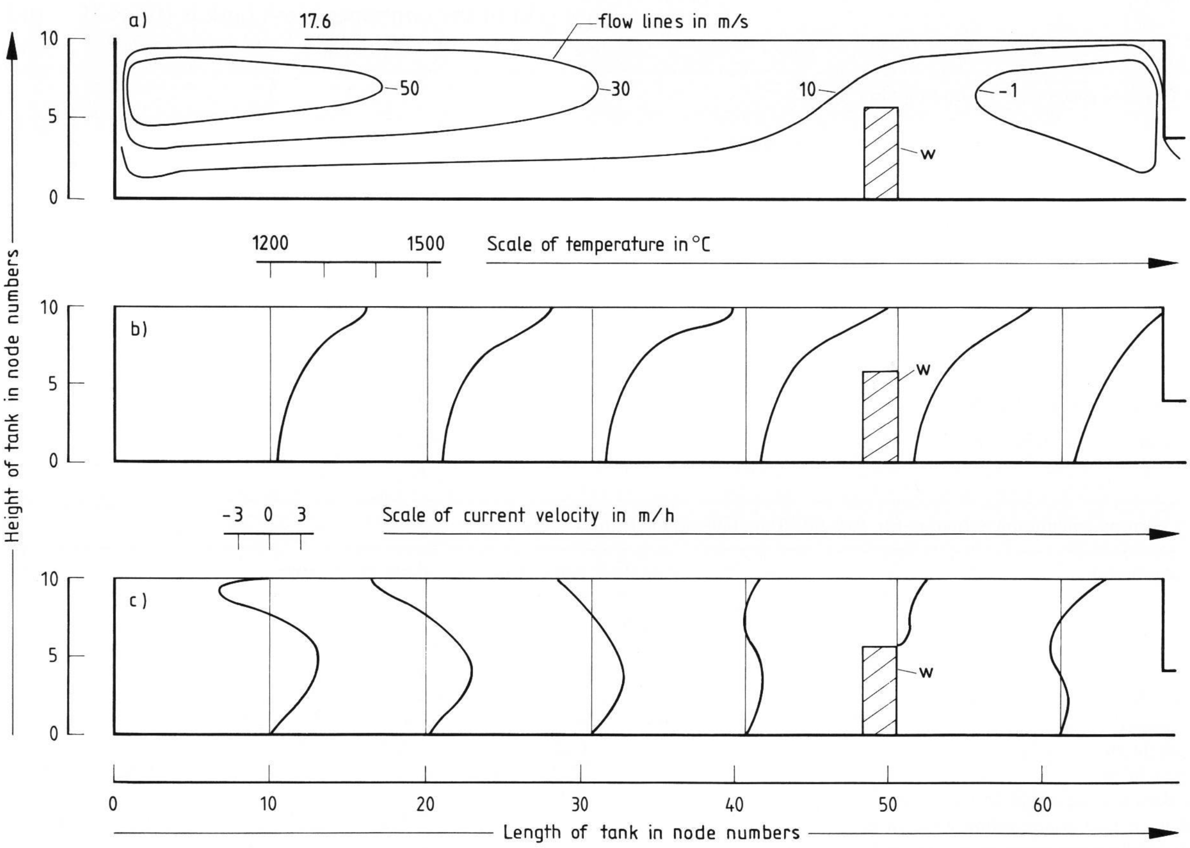
property	container glass [8]	float glass [9]
viscosity in Pa s	$\mu = 10^{-2.81 + 4571/(\vartheta - 253.4)}$	$\mu = 0.1 \cdot 10^{-1.79 + 4737/(\vartheta - 230)}$
dimensionless thermal conductivity	$\Gamma_{eff} = (k_0 + k_1\,\vartheta + k_2\,\vartheta^2) \cdot 418.68$	$\Gamma_{eff} = (k_0 + k_1\,\vartheta + k_2\,\vartheta^2 + k_3\,\vartheta^3) \cdot 418.68$
effective heat conductivity in W/(m K)	$k_0 = 0.9996$ $k_1 = -1.044 \cdot 10^{-2}$ $k_2 = 2.085 \cdot 10^{-5}$ –	$k_0 = -1.532$ $k_1 = 4.464 \cdot 10^{-3}$ $k_2 = -4.489 \cdot 10^{-6}$ $k_3 = 1.497 \cdot 10^{-9}$
density in kg/m³	$\varrho_0 = 2500$	$\varrho_0 = 2305$
specific heat capacity in J/(kg K)	$C = 1256$	$C = 1373.3$
cubical expansion coefficient in 1/K	$\beta = 5 \cdot 10^{-5}$	$\beta = 5.95 \cdot 10^{-5}$

Table 2. Various calculating schemes for two different glass tank furnaces

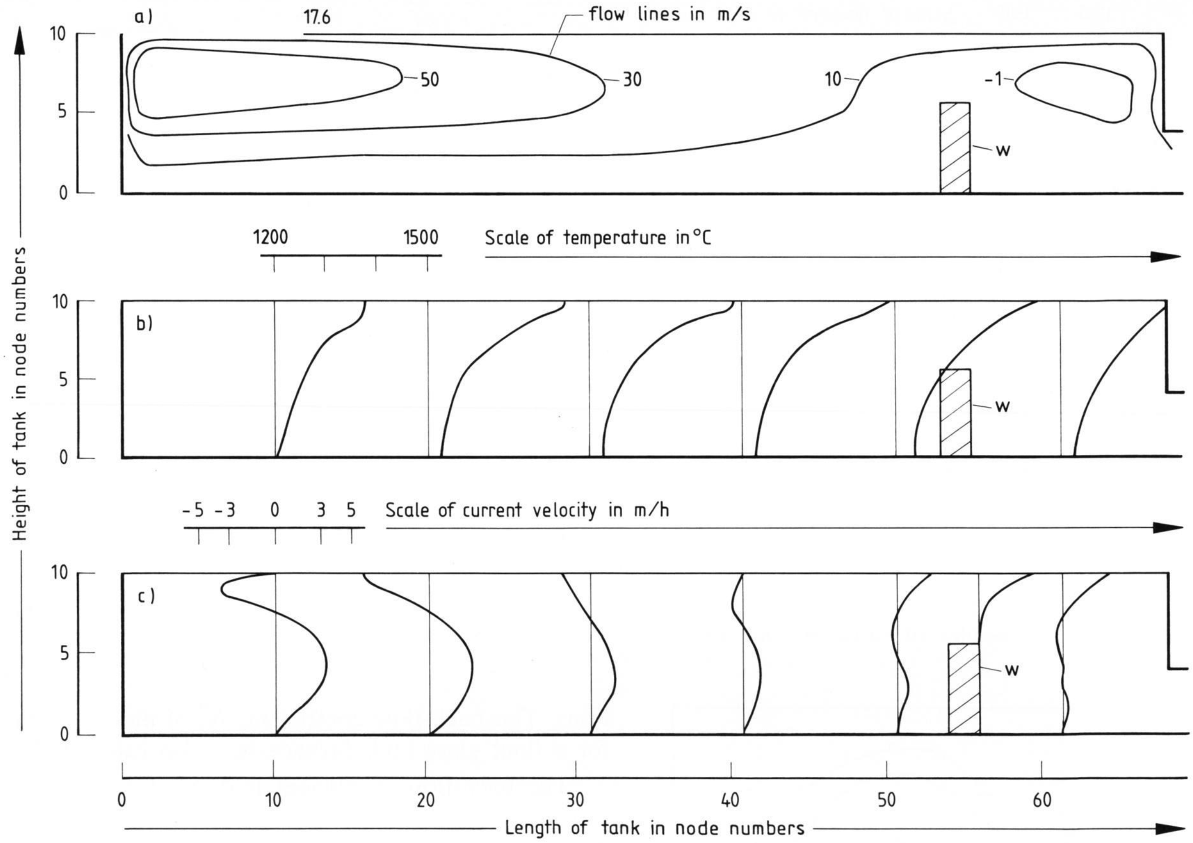
furnace parameter	container glass tank			float glass tank				
	scheme no.							
	1	2	3	4	5	6	7	8
pull in t glass/d	70	70	70	250	250	250	250	250
tank depth in m	0.85	0.85	0.85	1.5	1.5	1.2	1.5	1.2
weir height in m	–	0.51	0.51	–	–	–	0.6	0.5
distance between weir and throat in m	–	3.2	2.4	–	–	–		
depth of neck and conditioning zone in m	–	–	–	1.2	1.2	1.0	1.2	1.0
depth of conditioning zone in the second half of the rear in m	–	–	–	–	1.0	0.8	–	0.8



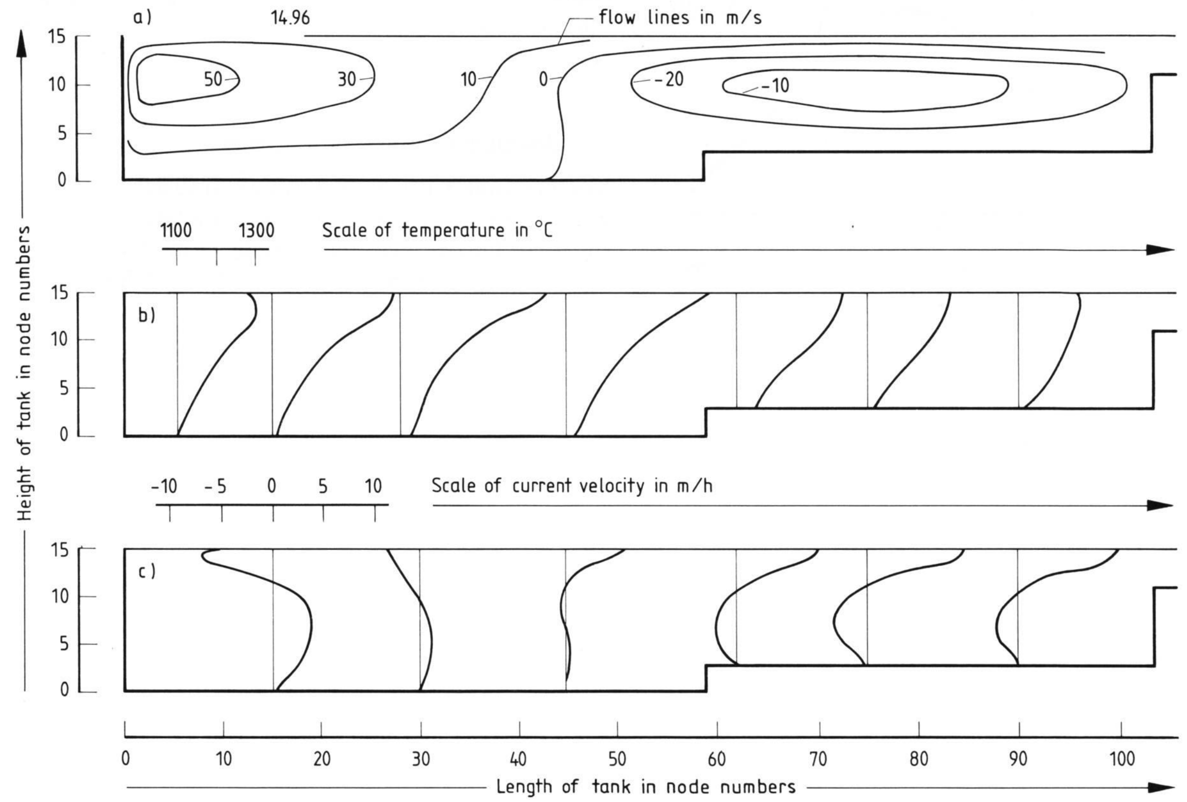
Figures 3a to c. Results of calculations after scheme no. 1 for a container glass tank without a weir; a) flow lines with a surface flow line number of 17.6, b) temperature profiles, c) current velocity profiles.



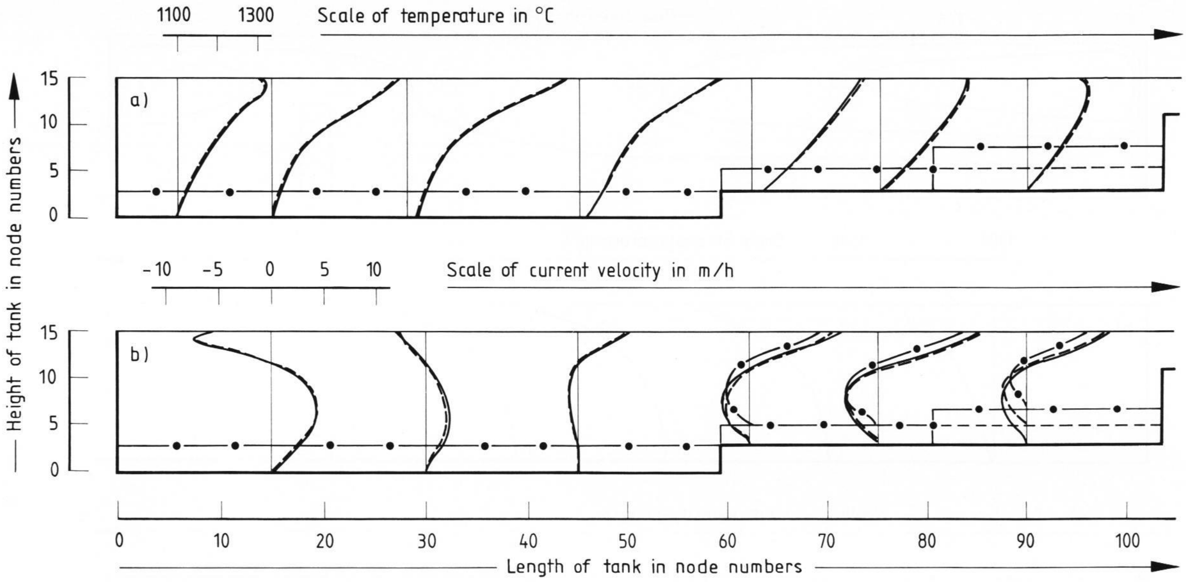
Figures 4a to c. Results of calculations after scheme no. 2 for a container glass tank with a 0.51 m high weir (w); flow lines with a surface flow line number of 17.6, b) temperature profiles, c) current velocity profiles.



Figures 5a to c. Results of calculations after scheme no. 3 for a container glass tank with a 0.51 m high weir (w); a) flow lines with a surface flow line number of 17.6, b) temperature profiles, c) current velocity profiles.



Figures 6a to c. Results of calculations after scheme no. 4 for a float glass tank without a weir; a) flow lines with a surface flow line number of 14.96, b) temperature profiles, c) current velocity profiles.



Figures 7a and b. Comparison of computation results after schemes no. 4 (———), 5 (– – –) and 6 (– · –) for a float glass tank with a 0.6 m high weir; a) temperature profiles, b) current velocity profiles.

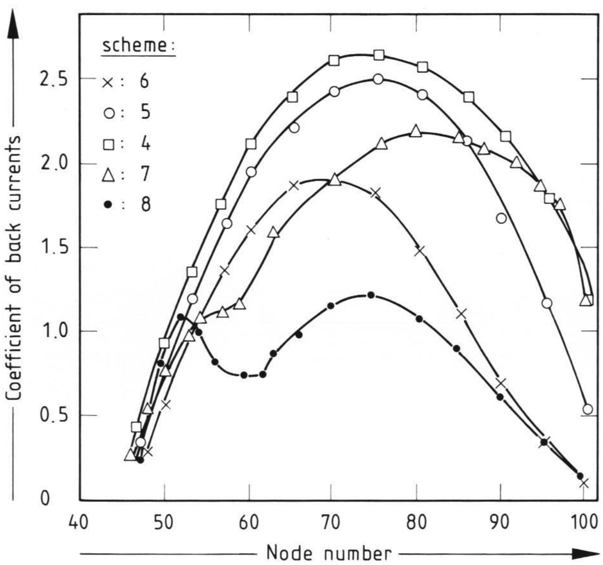


Figure 8. Coefficient of back currents as a function of node numbers (length direction) for various schemes.

that of float glass tank is (103 × 15) node numbers. Owing to the coupling of equations (11 to 13), they must be solved simultaneously. The iteration covers a) temperature, b) vorticity, c) stream function, d) repeat from step a). In order to ensure stability of the numerical solution, the convective terms are calculated by the "upwind" difference [7]. The relaxation method is used in the calculation. The relaxation factors are 0.08 (for calculating Ω) and 1.3 (for calculating θ and ψ). Calculating programmes were worked out by the authors using the computer language ALGOL [8]. A Burroughs-6935 computer was employed for the calculation, which yielded the numerical solution of temperature and velocity profiles of molten glass for different tank constructions. The back-flow coefficient, N, of molten glass for a float glass tank furnace was also calculated.

The back-flow coefficient is defined as follows:

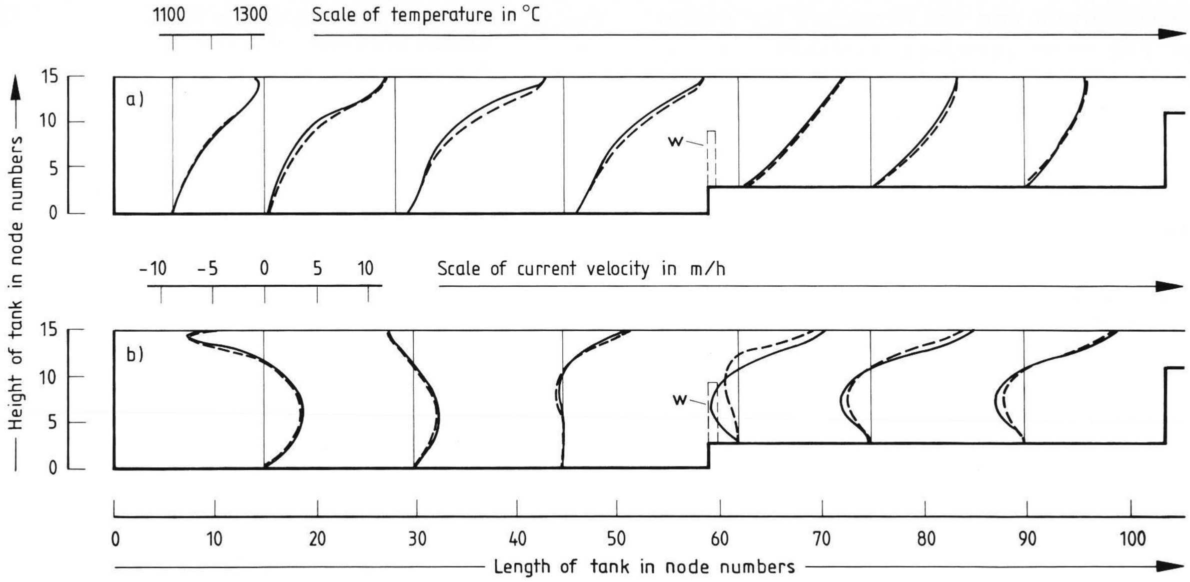
$$N = V_b/V_0$$

where V_b is the volume of the back flow, and V_0 is the volume of the furnace pull. The physical properties of container glass and float glass are given in table 1; for the calculating schemes see table 2; for the calculating results see figures 3 to 10.

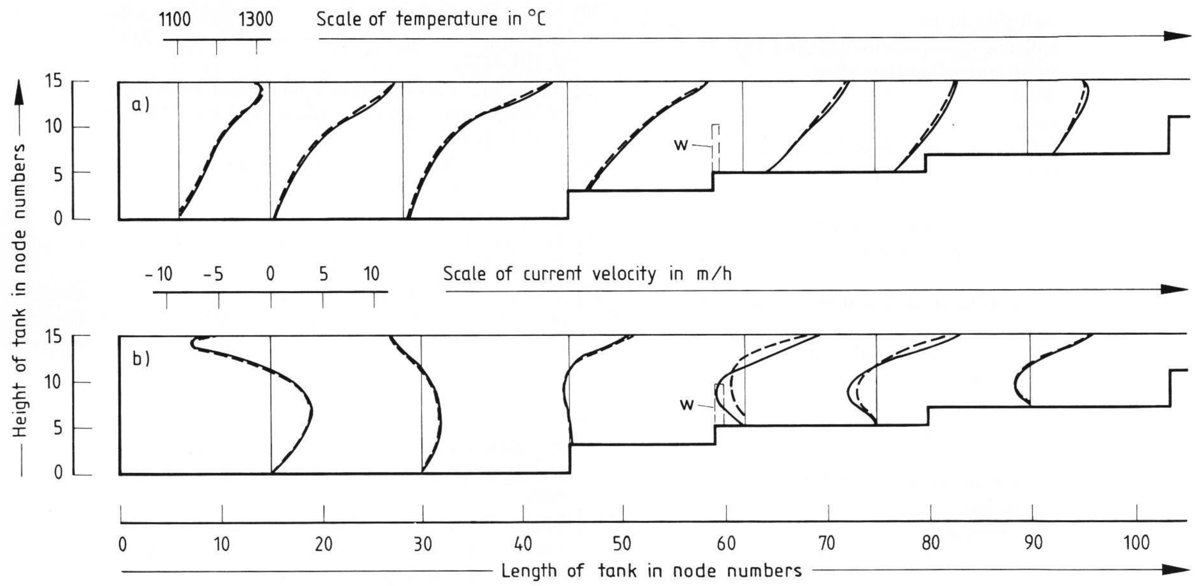
5. Discussion and conclusions

The phenomena of flow and heat transfer of molten glass in tank furnaces can be well simulated, although the two-dimensional mathematical model neglects the effect of transverse glass flow. The results obtained from the mathematical model were in good agreement with those obtained from the physical model and the measurements on a full-scale tank furnace (figure 11) [2 and 10].

Figures 3a to c show that in the container glass tank, the glass flowing towards the trough of the refining zone not only has the upstream, but also the deep downstream close to the bottom of tank. The installation of a weir (figures 4a to c, 5a to c) can prevent the deep downstream from going to the refining area, thus obtaining a glass of good quality, and decreasing the current back to the hot spot from the front wall. A weir being too close to the throat considerably increases the velocity of the surrounding glass. This effect corrodes and damages the weir. If the weir is too far from the throat it loses its function. The height of the weir will be most favorable when the glass is much refined and the back current is



Figures 9a and b. Comparison of computation results after schemes no. 4 (———) and 7 (– – –) for a float glass tank; a) temperature profiles, b) current velocity profiles.



Figures 10a and b. Comparison of computation results after schemes no. 4 (———) and 8 (– – –) for a float glass tank with a 0.5 m high weir; a) temperature profiles, b) current velocity profiles.

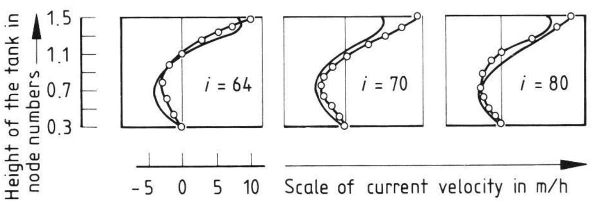


Figure 11. Comparison of computation results obtained from the mathematical and the physical model with the node number i as parameter. —○—○—: mathematical model (scheme no. 4), ———: physical model [7].

considerably decreased. Comparing schemes no. 2 and 3 shows that the optimum height of the weir is 3/5 of the glass depth, and the best distance between weir and throat is 1/4 of the tank length (i.e., scheme no. 2 is better than scheme no. 3).

Figures 6a to c show that a strong local convection exists in the glass melt, its flow is much higher than the pull. The back current going to the hot spot from the front wall will reheat the refined glass, thus, wasting a lot of thermal energy. Therefore, the glass tank furnace structure should be designed in such a way that the back current is decreased as much as possible, which will be significant for thermal efficiency increase and energy savings. For that purpose, the structure of the tank and the bottom of the conditioning zone were altered, and a weir was installed at the neck (see table 2). Figures 7 to 10 show that the decrease of the depth of the conditioning zone and the installation of the weir at the neck will considerably decrease the back flow of the glass. The shallower the conditioning zone is, the less is the back current and the better are the energy savings. The most promising possibility for decreasing the back current is a tank which has the step-shape bottom combined with a weir at the neck (scheme no. 8).

6. Symbols

A	dimensionless thermal diffusivity of glass
A_c	constant
a	thermal diffusivity of glass at temperature ϑ in m^2/s
a_0	thermal diffusivity of glass at temperature ϑ_{min} in m^2/s
B_c	constant
C	specific heat capacity in J/(kg K)
F_B	rate of batch feeding in m/s
g	gravitational acceleration in m/s^2
h	depth of molten glass in the tank in m
h_c	exit height of the float glass tank or throat height of the container glass tank in m
I_c	constant
i	node number
k_0 to k_3	constants
L_B	length of batch pile in m
M	constant calculated by equations (15 and 16)
N	back-flow coefficient of molten glass
p	pressure in Pa s
Ra	Rayleigh number
u	component of velocity in x direction in m/h
V_0	volume of furnace pull in m^3/h
V_b	volume of back flow in m^3/h
v	component of velocity in y direction in m/h
v_B	melting velocity of batch in m/h
W	width of tank in m
X	dimensionless coordinate in longitudinal direction
X_c	$L_B/2$ in m
x	coordinate in longitudinal direction in m
Y	dimensionless coordinate in depth direction
Y_c	constant
y	coordinate in depth direction in m
β	coefficient of cubical expansion in 1/K
η	dimensionless viscosity
θ	dimensionless temperature
ϑ	temperature in °C
ϑ_c	constant
ϑ_{max}	maximum temperature of glass in °C
ϑ_{min}	minimum temperature of glass in °C
Γ	dimensionless thermal conductivity
λ_{eff}	effective thermal conductivity in W/(m K)
μ	viscosity in Pa s
ϱ	density of glass in kg/m^3
ϱ_0	density of glass at temperature ϑ_{min} in kg/m^3
ϱ_B	density of batch in kg/m^3
Ψ	dimensionless stream function
ψ	stream function in m^2/s
Ω	dimensionless vorticity
ω	vorticity in 1/s

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