Supplementary Material

"Spatial self-organization of macroscopic quantum states of exciton-polaritons in acoustic lattices"

J. V. T. Buller¹, R. E. Balderas-Navarro², K. Biermann¹, P. V. Santos¹ and E. A. Cerda-Méndez^{1,2,3*} ¹ Paul-Drude-Institut für Festkörperelektronik, Hausvogteiplatz 5-7, 10117 Berlin, Germany

² Instituto de Investigación en Comunicación Óptica, Universidad Autónoma de San Luis Potosí, Av. Karakorum 1470, Lomas 4ª Secc, 78210, San Luis Potosí, México

³ Instituto de Física Universidad Autónoma de San Luis Potosí, Av. Manuel Nava #6, Zona Universitaria, C.P. 78290 San Luis Potosí, México

In this Supplementary Material section, we discuss, in section I, the analysis made to determine the Gaussian-like dependence of the square modulus of the polariton gap soliton (GS) to establish the characteristic length $\ell = 2\pi/\Delta k$, where Δk is the full-width at half-maximum of the k-space emission. In section II, we discuss the mechanisms that play a role in the limitation of the growth of the size of the gap soliton (GS) wave function when the number of injected particles is increased. Namely, the blue-shift of the dispersion by polariton-polariton interactions in combination with the GS reaching its natural size determined by the interaction energy gN (where g is the interaction constant and N is the particle density) and its effective mass, and the penetration of the GS into the lattice gap when increasing the number of particles.

I. Spatial dependence of $|\Psi|^2$

The simple formulation we use in the main text to define the characteristic length ℓ parameter is valid for a Gaussian-like dependence of the coherence on distance, which translates into a Gaussian line shape for the k-space peaks. We assumed this dependence based on the analysis of our data, as is shown in the plots in Figure SM1.



Figure SM1. Left comun: k-space dispersions at low and high optical pump power $P_L=P_{th}$ and $P_L=10P_{th}$, where Pth is the threshold of formation of the polariton macroscopic quantum state. The intensity profiles of each spectra can be fitted using simple Gaussian functions. Right column: real-space tomograms at the energy of the maximum intensity of the k-space spectra. The lower plots are intensity profiles taken at different lines over the spot (colored lines). The envelope of the intensity (red line) is a simple Gaussian curve.

The left panels show spectra in k-space and the right panels show tomograms in real space at the energy of the maxima of intensity of the k-space spectra. In each case we show two images, one at low

laser excitation power P_L (close to threshold P_{th}), and the other one at high excitation power (around 10 x threshold).

The profile of the k-space peaks can be nicely fitted with Gaussian curves ("k-space" column). Also, the intensity profiles of the real space tomograms, which are shown in the lower panels of the "Real Space" column, can be fitted with Gaussian curves. The several curves under the Gaussian profile (colored lines) correspond to lines taken from the tomogram image at different vertical positions of the emission spot. This tells us that there is a Gaussian-like spatial-dependence of the MQP wave function.

Finally, in a former article of ours¹, we realized direct measurements of the coherence length of a polariton MQP modulated by a single SAW by means of interferometry. The main result is shown in the next figure. The left panel is a spatial map ($30 \times 30 \mu m^2$) of the interference contrast of the modulated MQP emission, which is directly proportional to the coherence length of its wave function. An intensity profile of the contrast map taken along the white line (right panel) reveals that the spatial dependence of the coherence length of the polariton MQP wave function under the SAW modulation can also be fitted with a good approximation using a Gaussian curve.



Figure SM2. Left comun: a) Intereference contrast spatial map ($30 \times 30 \mu m^2$ area) of a polariton MQS modulated by a single SAW propagating in the y direction. B) Intensity profile taken along the white line in a), which has a Gaussian intensity envelope (red curve).

In summary, our experimental evidence points to a Gaussian spatial dependence of the intensity and of the coherence length of $|\Psi|^2$ of the polariton GS. The, to the FWHM of the k-space peaks emission can thus be assigned what we call a "characteristic length" ℓ , which could be related the coherence length of the GS wave function. However, a direct measurement of the coherence length (such as the one we present in the second figure) in direct confrontation with the FWHM of the k-space peaks is needed to absolutely confirm this relation.

II. Mechanisms that limit the growth of the gap soliton size.

The structure of the polariton dispersion under the SAW modulation (and also that of the pump states) is modified and presents a rich structure, as is shown in Fig. SM3. This means that the phase-matching rules that determine the so-called "magic angle" in the optical parametric oscillator are strongly relaxed. The laser energy is 1.5353 meV, as indicated in Section II (Methods) of the main text. Due to the negative detuning of the sample, the dispersion curvature is pronounced, so the bottom of the SAW-modulated dispersion at k=0 is at 2.4 meV below the laser energy (the Rabi splitting in the sample is larger than 6 meV). The spectrum of the polariton dispersion in Fig. SM1 shows, for example, that for k-vectors close to that of the pump laser $k_{laser} = 1.7 \ \mu m^{-1}$, the dispersion shows replicas at higher energy. (The red dot in the figure shows the approximate position of the laser, whose reflection is blocked during the experiment).



Figure SM3. SAW-modulated polariton dispersion. The red dot indicates the pump laser, whose reflection is blocked during the experiment.

It is thus expected that the emission of the polariton GS show a complex behavior as a function of the optical excitation density as the pump state goes off and in resonance with the modified polariton dispersion modulated by the SAW. For example, the dispersion can be blue-shifted due to interactions and go off resonance with the laser only to find a second resonance with a dispersion replica at lower energy at higher laser power density.

In Figure SM4, we plot the dependence of the emission intensity of the gap soliton as a function of the excitation optical density. One can see that around PL=50-100 μ W/ μ m² there is a plateau in the emission intensity of the gap soliton, which may be caused by the dispersion getting off resonance with the laser due to blue-shift by interactions. Since it is off-resonance, no more polaritons can be injected into the GS, so its emission intensity stops increasing.

For $PL \ge 300 \ \mu W/\mu m^2$, the intensity of emission of the GS starts increasing again, which indicates that more polaritons are injected into the GS. This might well happen due to the laser finding a second resonance with a lower energy dispersion replica at $k=k_{laser}$. Note that this increase in intensity, which indicates a larger number of particles, coincides with the slight decrease of the GS size in the plot shown in the main text. This is consistent with the picture of the GS being pushed into the lattice bandgap by an increase in the number of particles, which further contributes to its localization. One must, however, take also into account the screening of the the SAW-induced lattice by the interactions, which reduces the effective mass.



Figure SM4. Dependence of emission intensity of the gap soliton with the optical power density of the pump.

In conclusion, we have phenomenologically discussed how the possible mechanisms that limit the size of the GS occur in as the optical pump power is increased. These mechanisms are the blue-shift of the dispersion by inter-particle interactions, the GS reaching its natural size determined by interaction

energy gN, effective mass, and the potential screening and GS localization due to penetration into the gap when more particles are injected. The full description of the mechanics of the system taking into account all of these processes requires, however, an elaborate theoretical model which is left for future work.