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## Mutual interactions of magnetized particles in complex plasmas

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**Abstract.** Various mutual dust–dust interactions in complex plasmas, including the forces due to induced magnetic and electric moments of the grains are discussed. It is shown that the dipole short-range forces can be responsible for the formation of field-aligned chains. Such chains may incorporate a few tens of individual particles, as frequently observed in experiments.

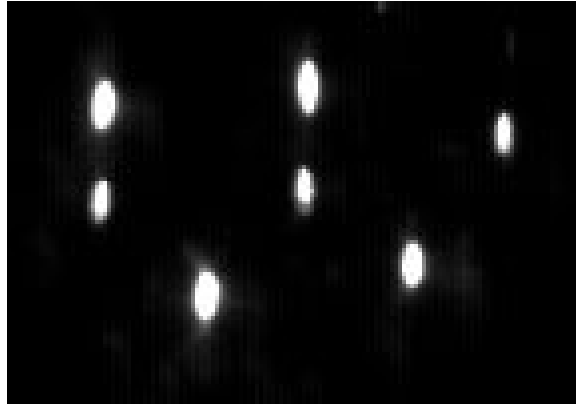
### Contents

1. <b>Introduction</b>	1
2. <b>Binary interactions of magnetized particles</b>	2
3. <b>Formation of particle chains</b>	5
4. <b>Conclusions</b>	7
<b>Acknowledgments</b>	8
<b>References</b>	8

### 1. Introduction

In the last decade, complex plasmas have attracted great interest as a newly developing field in plasma physics and because of the many interactions with other research areas, such as

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**Figure 1.** Agglomerated magnetic particles suspended in the discharge. The magnetic field is vertical with the gradient pointing up. The agglomerates are elongated in the vertical direction. The field of view is  $2.1 \times 1.48$  mm.

strong coupling phenomena, colloid physics, environmental research, astrophysics, plasma processing technologies etc. In fundamental physics, the most surprising discovery has been the observation of crystal-like structures which spontaneously form when microparticles are trapped in a sheath electric field, (so-called plasma crystals, [1]–[4]). These complex plasma structures are usually generated in parallel plate rf discharge plasmas using monodisperse plastic particles of micrometre size trapped in the sheath electric field of the lower electrode [2, 3, 5]. The particles charge up in response to the inflow of plasma electrons and ions (negatively) because of the large mobility of electrons. In the sheath region, the gravitational force on the particles can be balanced by the electric force and the particles levitate. Due to their mutual Coulomb repulsion, the dust particles can form ordered structures like plasma crystals.

Recently, multilayer complex plasma structures have been studied in an external magnetic field [6]. In these experiments, spherical micrometre sized paramagnetic particles are levitated in the sheath region (figure 1). The levitation increased with the strength of the magnetic field. Moreover, some particles attracted each other and formed elongated structures—grain chains, oriented vertically, parallel to the field lines of the external magnetic field. These features cannot be explained on the basis of pure electrostatic forces. Short-range dipole interactions between magnetized grains have to be invoked. Hence there is a need for a comprehensive treatment of various mutual dust–dust interactions, including the forces due to induced magnetic and electric moments of the grains. Which of the forces prevail is determined not only by the parameters of the dust particles, but is dependent on the discharge plasma conditions and magnitudes of the sheath electric and magnetic fields as well.

Our paper is structured as follows, in section 2 we discuss binary interactions between magnetized and polarized grains, as a prerequisite for particle agglomeration, before turning in section 3 to the formation of elongated structures. Finally, our conclusions are given in section 4.

## 2. Binary interactions of magnetized particles

We assume that there is a balance between the main bulk forces acting on dust grains, which provide their levitation in the plasma, e.g. the gravitation is balanced by either the electric force

due to a sheath electric field  $E_0$ ,

$$Mg = QE_0, \quad (1)$$

or the magnetic force associated with a vertical gradient of an external magnetic field  $B_0$ ,

$$Mg = -m \frac{\partial B_0}{\partial z}. \quad (2)$$

Here  $M$ ,  $Q$  and  $m$  are mass, charge and magnetic dipole moment of the grain, respectively and  $z$  denotes a vertical coordinate. To investigate mutual interactions between the particles, we need to take into account long-range as well as short-range forces. Regarding the long-range forces in a dusty plasma, it is tacitly agreed that they are electrostatic. Indeed, any two grains with like charges  $Q_1$  and  $Q_2$  separated by a distance  $d$  are subject to the repulsion force

$$F_Q = \frac{Q_1 Q_2}{d^2} \exp\left(-\frac{d}{\lambda_D}\right), \quad (3)$$

where  $\lambda_D$  is the plasma Debye length.

The situation is different if we consider the short-range dipole interactions. Indeed, a spherical particle with magnetic permeability  $\mu$ , immersed into a magnetic field  $B_0$ , gets a magnetic moment [7]

$$\mathbf{m} = a^3 \frac{(\mu - 1)}{(\mu + 2)} \mathbf{B}_0, \quad (4)$$

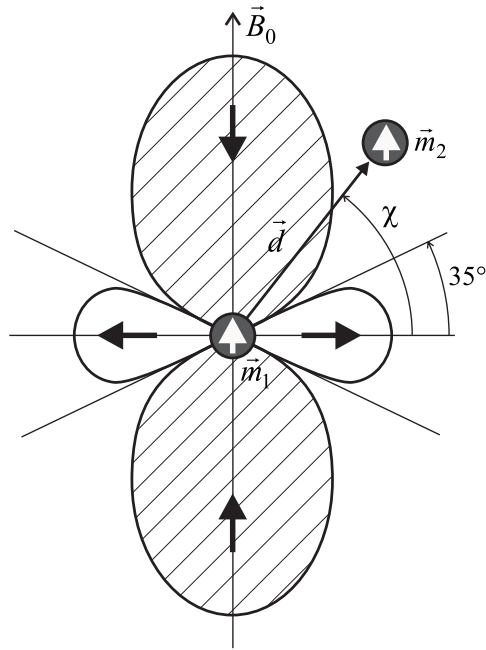
where  $a$  is the grain radius. First, consider the interaction of two magnetized particles at a relatively large separation ( $d > a$ ). The dipole moments  $\mathbf{m}_1$  and  $\mathbf{m}_2$  are both parallel to the external magnetic field  $B_0$  (figure 2). The force component along  $d$  is then given by [8]

$$f_m = \frac{3m^2}{d^4} (1 - 3 \sin^2 \chi), \quad (5)$$

where the angle  $\chi$  has been defined in figure 2. The force  $f_m$  can be either attractive or repulsive, depending on the relative positions of the dipoles. In figure 2 the  $f_m(\chi)$  dependence for two identical spherical particles is shown. The domain of angles corresponding to attraction is hatched. Dust grains moving along the same field line at different heights are attracted to each other and can form an extended chain, since the magnetic forces act in the same direction. In contrast, particles located at the same height are repelled when approaching one another and hence chains aligned perpendicular to the external magnetic field are impossible. The highest value of attraction for spherical grains corresponds to  $\chi = \pi/2$  and equals

$$f_{m,max} = 6B_0^2 a^6 d^{-4} (\mu - 1)^2 / (\mu + 2)^2. \quad (6)$$

It is usually assumed that magnetic moment effects in complex plasmas are very short-range and much weaker than the electrostatic interactions (3). However, for microparticles magnetized by an external magnetic field the condition  $f_{m,max} \sim F_Q$  can be easily realized in the laboratory even at relatively moderate magnitudes  $B_0$ . In order to give an idea of the magnitudes of  $B_0$  involved, we will give some simple numerical estimates, using complex plasma parameters in typical experiments on paramagnetic grains suspended in discharge plasmas [6]. We assume that dust particles of a size  $a = 4.5 \mu\text{m}$  and magnetic permeability of  $\mu \simeq 4$ , have an average charge  $Q \sim 10^4 e$  and are characterized by an intergrain distance  $d \sim (2-3)\lambda_D$ , with  $\lambda_D \sim 10^{-2} \text{ cm}$ . Such parameters imply that the dipole force (5) may dominate over the electrostatic force (3),



**Figure 2.** The interaction of magnetized particles of dipole moments  $m_1$  and  $m_2$ .

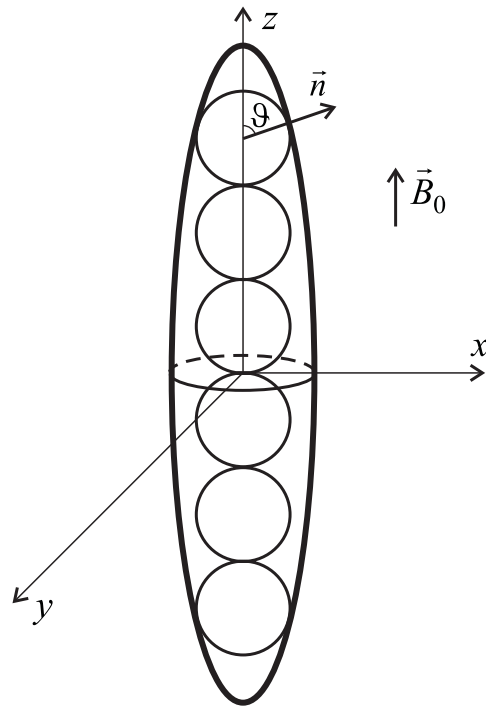
$f_{m,max}/F_Q \geq 1$ , if the magnetic field is of the order of  $B_0 \geq 20\text{--}50$  G. Such values can be easily achieved in laboratory conditions.

A spherical grain suspended in the sheath electric field  $E_0$ , becomes electrically polarized, in complete analogy with the magnetostatic case, acquiring a dipole moment,

$$\mathbf{p} = a^3 \frac{(\varepsilon - 1)}{(\varepsilon + 2)} \mathbf{E}_0, \quad (7)$$

where  $\varepsilon$  is the electric permeability of the particle material. Moreover, binary interactions of polarized grains are described by a force, which has exactly the form (5) of the magnetic interaction force, if the substitution  $m \rightarrow p$  is made. Once again this force may result in either repulsion or attraction according to the relative positions of the polarized particles. Since the sheath electric field  $E_0$  is directed vertically, the electric dipole force will act in the same manner as (5) and the maximum attraction force between the two vertically oriented dipoles can be calculated from  $f_{p,max} = 6E_0^2 a^6 d^{-4} (\varepsilon - 1)^2 / (\varepsilon + 2)^2$ .

The question whether or not the interaction between polarized grains is significant can be resolved numerically. Compare, for instance, the ratio  $f_{m,max}(\chi)/f_{p,max}(\chi)$ , which is specified by  $B_0/E_0$ . In experiments on the levitation of paramagnetic particles, the magnitude of the magnetic field  $B_0$  was rather high, up to  $(1\text{--}4) \times 10^3$  G [6], while the sheath electric field was  $E_0 \sim (10\text{--}100)$  V cm $^{-1}$ , so that  $B_0/E_0 \sim f_m(\chi)/f_p(\chi) \gg 1$ . Therefore, in this case we can neglect the weak electric polarization effects and focus only on the magnetic interactions (5) as the main reason for particle coagulation: particles moving along the same field line can join to form vertically aligned aggregates, provided the magnetic dipole attraction can overcome the electrostatic repulsion of likely charges. Then the magnetized particles could aggregate into long chains oriented along the external field  $B_0$ . Intuitively, such configurations can only be of limited extent. Equation (5) cannot be used for estimating this scale size, as it is valid for  $d \gg a$  (or at least  $d > a$ ), and it does not include particle coagulation.



**Figure 3.** A spheroid model of the coagulated particle structure in the direction of an external magnetic field.

### 3. Formation of particle chains

To estimate the possible length of particle chain structures, we consider a model of a prolate magnetized spheroid with semi-axis lengths along the  $x$ ,  $y$  and  $z$  directions,  $a = b$  and  $l \gg a$ , respectively (figure 3). Furthermore, we assume that its symmetry axis is parallel to the external field  $B_0$  ( $z$ -axis) and that the largest length of the grain structure  $l$  is sufficiently small so that a vertical gradient of the external magnetic field need not be taken into consideration. The force exerted by a magnetic field on a nonferromagnetic body is [9]

$$F_M = \frac{1}{4\pi} \oint \left[ \mathbf{B}(\mathbf{n}\mathbf{B}) - \frac{1}{2} B^2 \mathbf{n} \right] dS, \quad (8)$$

where the integral is taken over any surface enclosing the body. Here  $\mathbf{n}$  denotes the unit vector in the direction of the outward normal to the integration surface,  $\mathbf{B}$  is the magnetic field of the magnetized body near the surface and  $dS$  the surface element.

The highest attraction force appears in the middle of a particle chain. So imagine the spheroid to be separated into halves by a plane perpendicular to  $B_0$ . The slit between these halves is assumed to be infinitely narrow and we have to determine the force of attraction between the two halves by calculating the integral (8) over the surface of one half of the spheroid.

The magnetic field  $B^{(i)}$  inside the spheroid is uniform and equal to [9]

$$B^{(i)} \simeq \frac{B_0}{[1 + (\mu - 1)n_B]}, \quad (9)$$

where  $n_B = (1 - e^2)[\log(1 + e) \times (1 - e)^{-1} - 2e]/2e^3$  defines the depolarization factor along the external field and  $e = (1 - a^2/l^2)^{1/2}$  is the eccentricity. The field in the slit is perpendicular

to the surface and can be written as

$$\mathbf{B} = \mathbf{B}^{(i)} \simeq \frac{\mu \mathbf{B}_0}{[1 + (\mu - 1)n_B]}. \quad (10)$$

On the outer surface of the spheroid the field components are determined by

$$B_n = B_n^{(i)} = \frac{\mu B_0 \cos \vartheta}{[1 + (\mu - 1)n_B]}, \quad (11)$$

$$B_\vartheta = B_\vartheta^{(i)} = -\frac{B_0 \sin \vartheta}{[1 + (\mu - 1)n_B]}, \quad (12)$$

where  $\vartheta$  is the angle between the vectors  $\mathbf{n}$  and  $\mathbf{B}_0$ .

Substituting (10)–(12) in (8), we calculate the integral (8) along the surface of one half of the spheroid to obtain the highest attraction force due to the magnetic interaction between individual grains in the chain

$$F_M = \frac{B_0^2 a^2 (\mu - 1)^2}{16\pi [1 + (\mu - 1)n_B]^2 e^2} \left[ 1 + \frac{1 - e^2}{e^2} \log(1 - e^2) \right]. \quad (13)$$

Putting  $a \ll l$ ,  $e \simeq 1$  and  $n_B \ll 1$ , yields the attraction force

$$F_M \simeq \frac{B_0^2 a^2 (\mu - 1)^2}{16\pi}. \quad (14)$$

Opposing this attractive force are the Coulomb forces due to the like charges carried by individual grains in the chain. If these electrostatic ‘tensile’ forces exceed the attraction force (14), the chain may disintegrate into fragments (or will not form at all). Problems of this sort are often discussed in the context of the electrostatic disruption of charged grains, when repulsion of the like charges carried by different elements of the grain surface exceeds the strength of the grain material [10, 11]. So we can use the same approach.

The interaction force between charged bodies can be estimated from the variations of the system’s total electrostatic energy  $W$  during small virtual displacements of its parts. In a virtual displacement of a conductor along the coordinate  $\zeta$ , the  $\zeta$ -component of the force is  $f_\zeta = -\partial W / \partial \zeta$ . To apply this relation to a surface element  $\Delta S$ , let us assume the element to be displaced outward along the surface normal by a distance  $\Delta \zeta$ . Then the electrostatic energy in the space around the conductor will be reduced by  $\Delta W = (1/8\pi) E^2 \Delta S \Delta \zeta$ . The energy density,  $E^2/8\pi$ , can be expressed in terms of the surface charge density,  $\sigma = E/4\pi$ , so that the force acting on unit surface area may be written as

$$f_\zeta = \frac{1}{\Delta S} \frac{\Delta W}{\Delta \zeta} = 2\pi \sigma^2. \quad (15)$$

The force  $f_\zeta$  is independent of the cross-section position only for a spherical body, and the fracture can occur anywhere. Obviously, this is a consequence of the high symmetry. In general, the tensile force should be different at different cross-sections of an asymmetric body. For instance, the electric field in its vicinity is enhanced near points of highest curvature (especially near pointed tips if such are present). The surface charge density is enhanced in the same areas and, accordingly, so is the tensile force. Thus, tips will be the first to break away and the effect of electrostatic fractures is to remove surface irregularities and reduce the asymmetry.

To relate our spheroid model to (15), we need to introduce some assumptions about the dust particles, such as the conducting properties of the grains. In fact, any finite value of the material conductivity ultimately will tend to homogenize the potential on the grain surface,

ousting the electric field from its interior. In addition, we assume that the total charge of the spheroid can be considered formally as a sum of the charges of all individual grains of the chain, i.e.  $Q_0 = NQ = lQ/a$ . Then the charge density at the surface of the spheroid can be written as [12]

$$\sigma = \frac{Q_0}{4\pi abl} \left( \frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{l^4} \right)^{-1/2}. \quad (16)$$

The charge densities at three extreme points of the spheroid along  $x$ ,  $y$  and  $z$  then satisfy the inequality

$$\sigma_a \Big|_{\substack{x=a \\ y=0 \\ z=0}} = \sigma_b \Big|_{\substack{x=0 \\ y=a \\ z=0}} = \frac{Q}{4\pi a^2} \ll \sigma_l \Big|_{\substack{x=0 \\ y=0 \\ z=l}} = \frac{l}{a} \frac{Q}{4\pi a^2} \quad (17)$$

and, indeed, the charge density is highest near the sharper end of the spheroid and lowest near the obtuse one.

Once again imagine a spheroid cut by a plane at some distance  $z$  from its centre. The breakaway force acting on the spheroid segment can be evaluated by projecting the normal to the surface component  $f_n = f_\zeta$  to the  $z$ -axis and integrating along the surface. Then, the highest magnitude of the disruption force is found to be acting on the outermost grain of the chain and can be estimated as

$$F_e \simeq \frac{l^2 Q^2}{4a^4}. \quad (18)$$

By comparing the values of the attraction (14) and repulsion (18) forces, we can estimate the critical length  $l_{cr}$  for the chain to remain stable,

$$l_{cr} \simeq \xi \frac{B_0 a^3 (\mu - 1)}{2\sqrt{\pi} Q}. \quad (19)$$

Here we have introduced a dimensionless coefficient  $\xi \leq 1$ , which compensates for some of the assumptions made, in particular the fact that we compare the highest ‘disruption’ force at the ends of the chain with the highest ‘cohesion’ force on its centre. The value for  $\xi$  has to be determined experimentally.

For the typical aforementioned plasma parameters, the chain length (19) becomes  $l_{cr} \sim 150\text{--}200 \mu\text{m}$ . Since the particle size is much smaller ( $a \sim 4.5 \mu\text{m} \ll l_{cr}$ ), formation of vertical chains is possible under the action of a magnetic force. According to our theory such chains might incorporate a few tens ( $N \sim l_{cr}/a \sim 35\text{--}45$ ) of particles. Experimental results so far suggest an upper limit of  $N \sim 25\text{--}35$  grains [6]. This gives a coefficient  $\xi \sim 0.6\text{--}0.9$  in expression (19), in good agreement with the predictions of our spheroidal model.

Note that the resulting critical scale (19) is proportional to the external magnetic field strength, the permeability of the particle material and exhibits a strong dependence on particle size. Moreover, the chain length is indirectly dependent on plasma discharge conditions (in particular the electron temperature) through the value of the charge carried by individual grains.

#### 4. Conclusions

Mutual interactions between microparticles in complex plasmas have been studied when effects of an external magnetic field become important. It is shown that the electromagnetic forces from particle magnetization may result in mutual repulsion as well as attraction. The analysis in

this paper was restricted only to dipole interactions, with particular interest focusing on particle coagulation. It was found that magnetized grains can coalesce, forming field-aligned chains. A model was developed to determine the length of these chains. Since the ‘disruptive’ electrostatic forces increase with the distance from the centre, whereas the ‘cohesion’ magnetostatic forces decrease, there is an intrinsic length scale for these particle chains. These findings were applied to recent complex plasma experiments with paramagnetic particles. Our theoretical estimations show good agreement with experimentally observed data.

Our results are of direct interest to laboratory studies of magnetized complex plasmas, indicating several new effects. In particular, the model predicts that the chain length will increase when the magnetic field is increased, when the permeability of the particle material is higher and the grain size is larger. In addition, our results could be of importance when studying dust particle agglomeration in astrophysical environments and for aerosol removal, cleaning and purification devices.

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