

1 General Information

DFG reference number: joined project Fi548-16 and DE 7360/3-1

Project number 440764520

Project title: Topological transport control of colloidal particles

Names of the applicants: Thomas Martin Fischer¹ & Daniel de las Heras²

Official addresses:

1 Experimental Physics X, Universität Bayreuth, 95440 Germany

2 Theoretical Physics II, Universität Bayreuth, 95440 Germany

Names of the cooperation partners:

Prof. Dr. Arno Ehresmann (University of Kassel)

Prof. Feliks Stobiecki (former head of the Department of thin films), Dr. Maciej Urbaniak, and Prof. Piotr Kuświk (head of the Department of thin films), Institute of Molecular Physics, Polish Academy of Sciences, Poznań, Poland.

Reporting period: from 08/2020 to 02/2024.

2 Summary

We have studied experimentally and with computer simulations the transport of magnetic particles on top of magnetic patterns. The motion is driven by either a modulation loop of the orientation of a uniform external magnetic field or by a drift force. The application of an adiabatic modulation loop of the direction of an external magnetic field to magnetic colloids or macroscopic magnetic particles on a periodic pattern offers unprecedented control over the motion and assembly of such colloids or particles. The motion is topologically protected since only those loops that wind around special orientations of the external field induce particle transport. The set of winding numbers around the special orientations is the topological invariant that protects the motion. The colloidal or macroscopic particles are sorted into topological classes and the transport of each class can be controlled independently and simultaneously with the other topological classes. The use of non-periodic patterns facilitates the transport of identical colloidal particles independently and simultaneously. The complexity of the loop can be imprinted in either the pattern or the modulation loop. In twisted magnetic patterns high mobility peaks of non-topologically driven particles emerge at non generic magic angles, but these mobility peaks in contrast to topologically driven systems are very fragile and can be easily destroyed via the analogue of an Anderson transition.

3 Progress Report

3.1 Background and objectives of the project

The project had three main goals:

- Unravel new topological features of driven colloidal transport by performing new types of experiments and computer simulations that are analogues inspired from the already understood electronic system.
- Develop simple polyglot control loops to simultaneously transport different classes of colloidal clusters into desired directions. Use these polyglot commands for parallel

computing with dipolar or patchy colloidal clusters.

- Looking for new types of emergent behavior that arises from the competition of self-propelled and externally driven motion above magnetic patterns.

3.2 Progression of the work

We think we have unraveled new scientific understanding with respect to all of the three goals. Rather than giving a chronological description of our work progress we will order our results along four major goals (3.1) that we followed within the project.

3.3 Presentation of the results achieved

Topologically nontrivial aspects of transport of electrons are a major field of solid state research. Since many of the topological aspects of electronic systems can be explained using a semi-classical description of band theory this means that the topological aspects of the electronic transport is not tight to the quantum nature of the electrons, but is mainly a result of the symmetry of the crystalline structure of the atomic background. Important ingredients leading to the topologically nontrivial behavior is the existence of a non-vanishing Berry curvature, Berry-phase or geometric phase. Electrons are waves or particles depending on the way you measure them. The research on topologically non-trivial transport in classical systems has to a major extent focused on the behavior of photonic, acoustic, gyroscopic waves and has therefore neglected the question of whether one might find topologically non-trivial transport of particles. The reason for this may lie in the fact that most of the electronic transport is described using homology groups that seem to be the natural description of the topological features of waves. A mathematical group related to homology groups are the homotopy groups.

In our research we found that those homotopy groups are the key to understand the topological aspects of particle transport. In our research electrons are replaced by magnetic colloids and the atomic background is replaced by a magnetic pattern. Colloidal particles are driven (or they actively walk) on top of those magnetic pattern when we apply a homogeneous external magnetic field the direction of which adiabatically changes along a loop (a closed curve).

3.3.1 Results to goal 1: New topological features of driven transport

Active versus passive topologically protected transport Paramagnetic colloidal spheres assemble to colloidal bipeds of various length in an external magnetic field. When the bipeds reside above a magnetic pattern and we modulate the direction of the external magnetic field, the rods perform topologically distinct classes of protected motion above the pattern [1]. The topological protection allows each class to be robust against small continuous deformations of the driving loop of the external field. We observe motion of the rod from a passive central sliding and rolling motion for short bipeds toward a walking motion with both ends of the rod alternately touching down on the pattern for long bipeds. The change of character of the motion occurs in form of discrete topological transitions. The topological protection makes walking a form of motion robust against the breaking of the non symmorphic symmetry. In patterns with non symmorphic symmetry walking is reversible. In symmorphic patterns lacking a glide plane the walking can be irreversible or reversible involving or not involving ratchet jumps (Fig. 1). Using different gauges allows us to unravel the active and passive aspects of the topological walks.

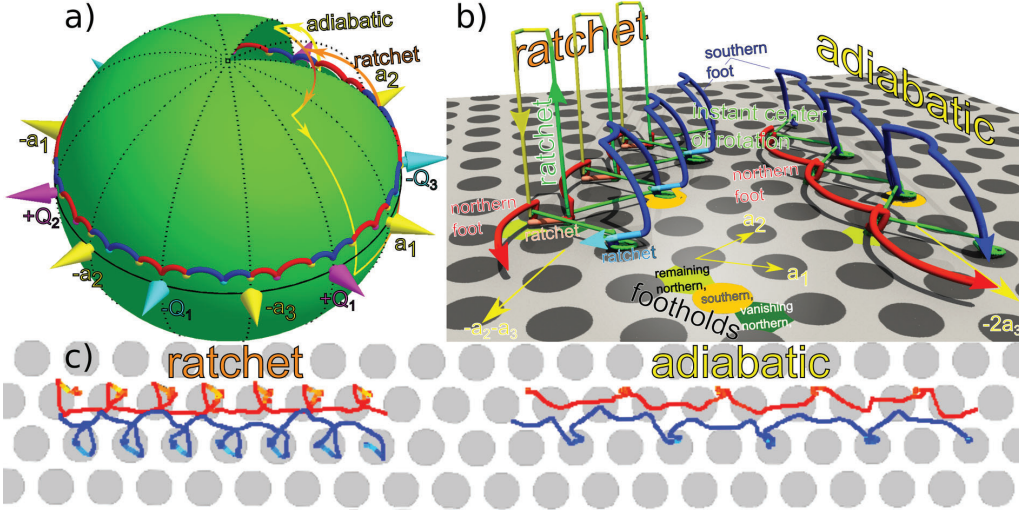


Figure 1: a) Control space and fence of a biped of length $b = 1.4a$ with an asymmetric control loop causing a ratchet and a symmetric control loop causing adiabatic transport. b) Simulated trajectories of the northern and southern foot and the instantaneous center of rotation of the same biped for the asymmetric and for the symmetric loop. In the ratchet trajectories brighter colors of the trajectories label the regions of higher speed when the ratchet jumps occur. c) Experimentally measured ratchet and adiabatic trajectories of the feet of the bipeds of lengths $b = 1.2a$ and $b = 1.2a$ respectively. The trajectories are colored with brighter colors where the velocity is higher.

Topologically protected transport of magnetic particles interacting via higher multi-pole interactions We simulate the trajectories of magnetic octupole colloids driven by periodic loops of an external magnetic field and placed above a two dimensional threefold symmetric magnetic pattern [2]. The octupoles avoid the threefold symmetric points above the lattice, either in a topologically trivial way, or by nontrivially winding around these high symmetry points both with respect to their position and with respect to their orientation . We calculate the full dynamical phase space of this winding behavior by changing both, lattice symmetries and modulation loops. We further use the nontrivial topology to braid (Fig. 2a) with octupoles and we supply a protocol for both braiding and weaving (Fig. 2b) with such microscopic particles. Our classical external field command should work equally well for quantum mechanical octupoles on magnetic nanopatterns, providing an explicit protocol for the exchange of anyonic quantum particles.

Geometric phases and dissipative phenomena in classical physics When adiabatic processes are non-topologically protected, then they are at least geometrical i.e. independent of the speed of the process. We report on the motion of a spinning sleeping top on an inclined plane [3]. Below a critical inclination angle the sleeping tops are force free. The trajectory of a sleeping top on weakly inclined planes in the adiabatic limit is invariant of the angular frequency of the top and thus invariant under a rescaling of the time, however not invariant under time reversal (Fig. 3). The stationary trajectory of the sleeping top is characterized by its Hannay type geometric angle to the in plane horizontal direction. At larger inclinations of the plane the stationary motion of the top becomes unstable and the top accelerates downhill. The behavior points towards a complex law of dry friction of the contact point between the top tip and the material of the inclined plane that depends on a slip parameter. We propose a phenomenological law of dry friction that

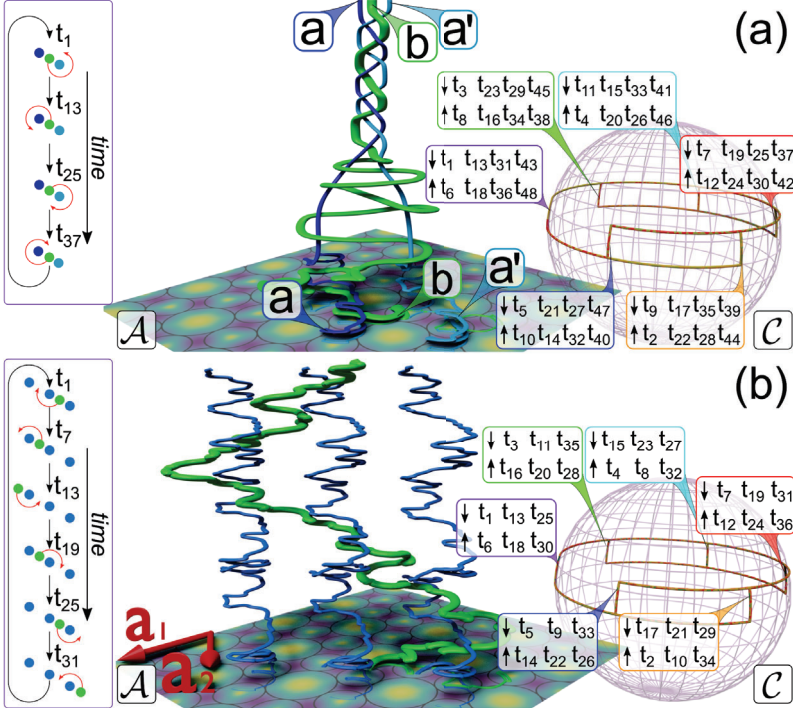


Figure 2: **a)** Trajectories of three octupoles forming a braid using two octupoles \mathbf{a} and \mathbf{a}' (blue) initially placed in equivalent potential minima of two different unit cells and a third octupole placed in a nonequivalent minimum \mathbf{b} (green). Virtual strings colored according to the corresponding octupoles indicate the movement of the octupoles in time. The braid can be seen at the top of the image, where we have deformed the trajectories into the standard braid representation without altering the topology. **b)** Space-time plot of the trajectories of a series of equivalent octupoles (blue) woven together by a fourth octupole (green) following a nonequivalent minimum position of the octupole potential. The complex modulation loop in control space \mathcal{C} used to drive the motion is shown on the right via the order of the transition times t_i , $i \in \{1, \dots, 36\}$ between the $N, w, w_\omega = (3, 0, 0)$ -phase and the $N, w, w_\omega = (2, -2, -1/2)$ -phase. The paths in action space \mathcal{A} are sketched by vanishing lines on top of the C_6 symmetric pattern. \mathbf{a}_1 and \mathbf{a}_2 are the lattice vectors of the pattern. The sketches on the left show the schematic exchange or winding of the particles.

can explain the relaxation of the top into the sleeping position, the geometric behavior of the top trajectories, and the instability of the stationary motion at larger inclination angles.

Adiabatic and irreversible classical discrete time crystals We simulate the dynamics of paramagnetic colloidal particles that are placed above a magnetic hexagonal pattern and exposed to an external field periodically changing its direction along a control loop [4]. The conformation of three colloidal particles above one unit cell adiabatically responds with half the frequency of the external field creating a time crystal at arbitrary low frequency. The adiabatic time crystal occurs because of the non-trivial topology of the stationary manifold. When coupling colloidal particles in different unit cells, many body effects cause the formation of topologically isolated time crystals and dynamical phase transitions between different adiabatic reversible and non-adiabatic irreversible space-time-crystallographic arrangements.

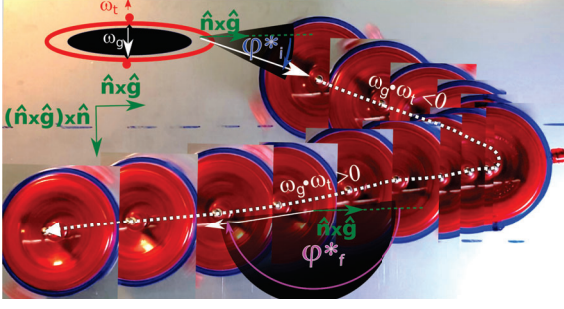


Figure 3: Trajectory of a top (red in the scheme) incorporating a counter rotating gyroscope (black in the scheme) having a steel tip and precessing on an inclined $\alpha = 0.07$ steel plate. The trajectory flips the direction from $\varphi_i^* \rightarrow \varphi_f^* = \pi - \varphi_i^*$ while the rotation frequency is slowly reversed by the loaded gyroscope. The behavior is in accordance with our phenomenological model. The independence of the travel direction of the magnitude of the top's angular frequency in both branches of the zig-zag trajectory points toward φ^* being a geometric angle.

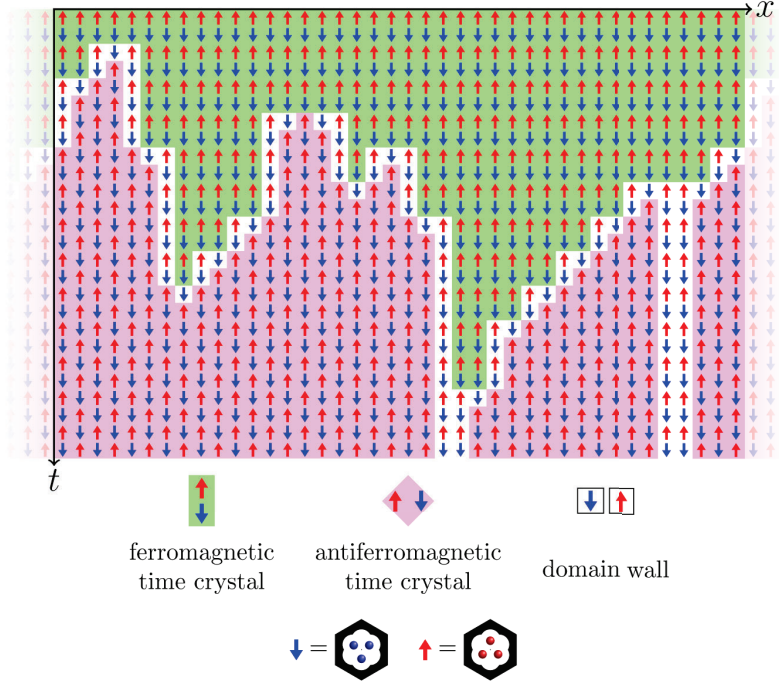


Figure 4: Stroboscopic development of a chain of honeycomb cells filled with three colloidal particles each. The initially ferromagnetic ordered time crystal undergoes a topological transition towards an antiferromagnetic time crystal. Space-time Wigner Seitz cells of the various orders are shown at the bottom. The spatial order in each cell is abbreviated by a blue or red arrow as indicated.

3.3.2 Results to goal 2: polyglot transport

Polyglot transport is the task of using a single control loop to simultaneously control the motion of different assemblies in equivalent or nonequivalent surroundings.

Simultaneous polydirectional transport of colloidal bipeds Detailed control over the motion of colloidal particles is relevant in many applications in colloidal science such as

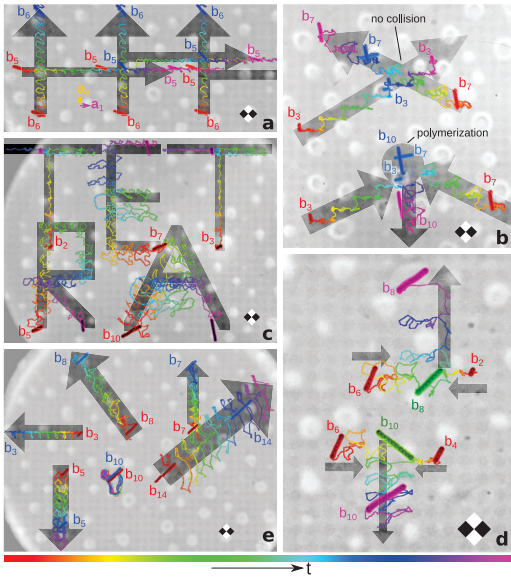


Figure 5: **Experimental trajectories of bipeds driven by parallel polydirectional loops.** **a** parallel didirectional loop of robustness $\rho = 0.16$ and compaction $c = 1/2$ transporting several bipeds b_5 into the \mathbf{a}_1 -direction and bipeds b_6 into the \mathbf{a}_2 -direction. **b** parallel tridirectional loop of robustness $\rho = 0.3$ and compaction $c = 7/8$ setting two bipeds b_3 and two bipeds b_7 on a collision course. One pair of bipeds collides and polymerizes to a longer biped b_{10} that is then transported into a third direction. The non colliding bipeds continue their motion. **c** Parallel tetradirectional loop of robustness $\rho = 0.2$ and compaction $c = 30/54$ commanding bipeds of five different lengths $\{b_3, b_7, b_2, b_5, b_{10}\}$ to simultaneously write four different letters. The biped b_2 is topologically equivalent to the biped b_3 and hence writes the same letter (T). **d** Parallel pentadirectional loop of robustness $\rho = 0.11$ and compaction $c = 5/6$ commanding a biped b_2 and a biped b_4 to polymerize with two colliding bipeds b_6 and then separate as they form a biped b_8 and a biped b_{10} . **e** Parallel hexadirectional loop of robustness $\rho = 0.06$ and compaction $c = 4/8$ transporting six bipeds of different lengths into six different directions. The color of both bipeds and trajectories indicates the time progress of one control loop (see bottom colorbar). A square region of the pattern of diagonal $2a \approx 22 \mu\text{m}$ is sketched in each panel to indicate the scale and the relative orientation.

lab-on-a-chip devices. Here [5], we use an external magnetic field to assemble paramagnetic colloidal spheres into colloidal rods of several lengths. The rods reside above a square magnetic pattern and are transported via modulation of the direction of the external magnetic field. The rods behave like bipeds walking above the pattern. Depending on their length, the bipeds perform topologically distinct classes of protected walks. We design parallel polydirectional modulation loops of the external field that command up to six classes of bipeds to walk on distinct predefined paths. Using such loops, we induce the collision of reactant bipeds, their polymerization addition reaction to larger bipeds, the separation of product bipeds from the educts, the sorting of different product bipeds, and also the parallel writing of a word consisting of several letters. Our ideas and methodology might be transferred to other systems for which topological protection is at work.

Simultaneous and independent topological control of identical microparticles in non-periodic energy landscapes Topological protection ensures stability of information and particle transport against perturbations. We explore experimentally and

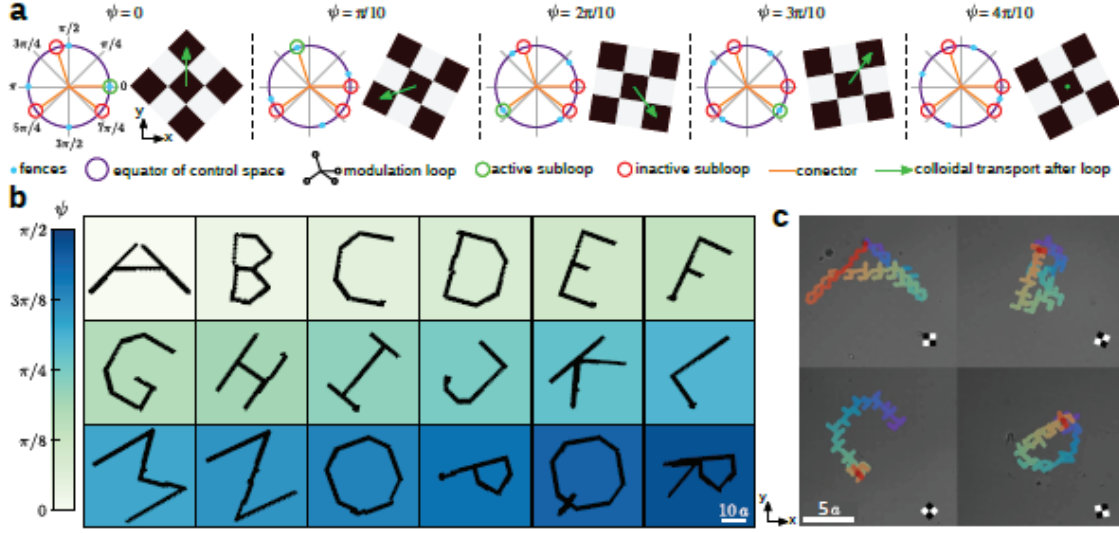


Figure 6: a) Five square magnetic patterns (and their corresponding control spaces) with a different value of the global orientation ψ , as indicated. The fences in \mathcal{C} (blue circles) are four points located on the equator (violet circle). The position of the fences depends on the value of ψ . The modulation loop consists of four interconnected subloops that wind counterclockwise. A subloop is active (green) if it winds around a fence point (blue circles) and inactive (red) otherwise. The orange segments of the modulation loop simply connect the different subloops. Depending on the value of ψ , the modulation loop induces different transport directions (green arrows) or no transport at all. b) A pattern made of 18 patches with square symmetry and different global orientation ψ (color bar). A modulation loop controls the trajectories of particles above each patch simultaneously and independently. The particle trajectories (black) write the first 18 letters of the alphabet. The length of the scale bar is $10a$. A movie can be found in Supplementary Movie 3. c) Experimental trajectories of colloidal particles above four square patches rotated with respect to each other. A schematic unit cell illustrating the global orientation is depicted in each patch. The length of the scale bar is $5a$ and in this case, we use patterns with $a = 7\mu m$. A unique modulation loop transports the four colloidal particles simultaneously. The trajectories are colored according to the time evolution from blue (initial time) to red (final time).

computationally the topologically protected transport of magnetic colloids above spatially inhomogeneous magnetic patterns, revealing that transport complexity can be encoded in both the driving loop and the pattern [6]. Complex patterns support intricate transport modes when the microparticles are subjected to simple time-periodic loops of a uniform magnetic field. We design a pattern featuring a topological defect that functions as an attractor or a repeller of microparticles, as well as a pattern that directs microparticles along a prescribed complex trajectory. Using simple patterns and complex loops, we simultaneously and independently control the motion of several identical microparticles differing only in their positions above the pattern. Combining complex patterns and complex loops we transport microparticles from unknown locations to predefined positions and then force them to follow arbitrarily complex trajectories concurrently. Our findings pave the way for new avenues in transport control and dynamic self-assembly in colloidal science.

3.3.3 Results to goal 3: Competition of self-propelled and externally driven motion

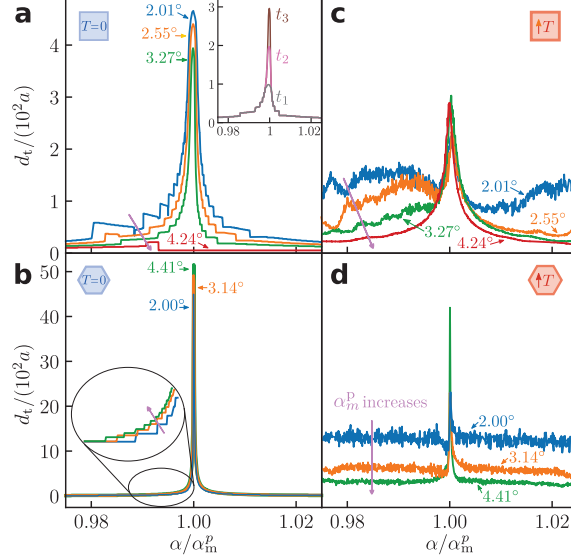


Figure 7: **Transport at magic and non-magic angles.** Average distance travelled by a particle as a function of the twist angle α (scaled with the magic twist angle α_m^p) at zero temperature (a,b) and finite temperature (c,d) in either square (a,c) or hexagonal (b,d) twisted patterns. Data sets for several magic angles are presented, as indicated. The magic angle increases in the direction of the pink arrows. The travelled distance is obtained by averaging the motion of 100 particles located initially at the origin (axis of rotation of the patterns) and driven by a drift force (acting during 100τ) of magnitude $f_d a/\varepsilon = 10$ in the square (a,b) and $f_d a/\varepsilon = 80$ in the hexagonal (c,d) patterns. The temperature is $T = 0$ in (a) and (b), $k_B T/\varepsilon = 0.3$ in (c) and $k_B T/\varepsilon = 0.8$ in (d). The drift forces and temperatures used here are indicated with colored arrows in Fig. ???. The inset in (a) shows data for a drift force acting during $t_1 = 25\tau$, $t_2 = 50\tau$, and $t_3 = 75\tau$ (at $T = 0$ and $\alpha_m \text{ sup sq} = 3.27^\circ$).

Enhanced colloidal transport in twisted magnetic patterns and in twisted lattices of optical tweezers Bilayers of two-dimensional materials twisted at specific angles can exhibit exceptional properties such as the occurrence of unconventional superconductivity in twisted graphene . We demonstrate here that novel phenomena in twisted materials emerges also in particle-based classical systems [7]. We study the transport of magnetic colloidal particles driven by a drift force and located between two twisted periodic magnetic patterns with either hexagonal or square symmetry. The magnetic potential generated by patterns twisted at specific magic angles develops flat channels, which increase the mobility of the colloidal particles compared to that in single patterns. We characterize the effect of the temperature and that of the magnitude of the drift force on the colloidal mobility. The transport is more enhanced in square than in hexagonal twisted patterns. Our work extends twistrionics to classical soft matter systems with potential applications to lab-on-a-chip devices. We also simulate the transport of colloidal particles driven by a static and homogeneous drift force, and subject to the optical potential created by two lattices of optical tweezers [8]. The lattices of optical tweezers are parallel to each other, shifted, and rotated by a twist angle. Due to a negative interference between the potential of the two lattices, flat channels appear in the total optical potential. At specific twist angles, known as magic angles, the flat channels percolate the entire system and the col-

loidal particles can then be transported using a weak external drift force. We characterize the transport in both square and hexagonal lattices of twisted optical tweezers.

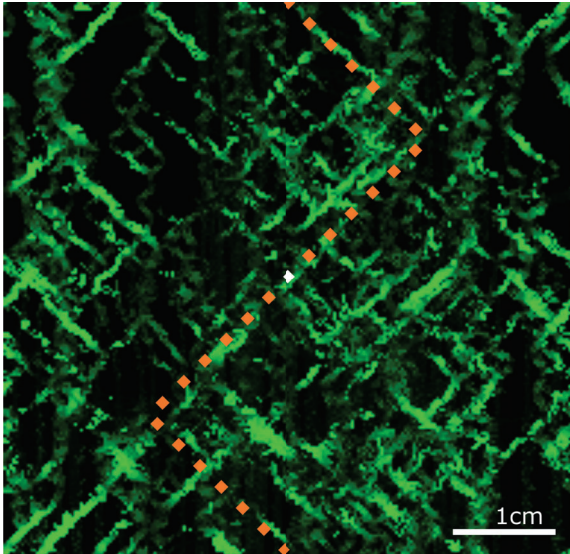


Figure 8: **Flatness parameter** folded into the twisted Wigner Seitz cell $\mathcal{W}_{\mathcal{T}\mathcal{W}}$. Green colors depict large flatness parameter and low flatness parameters are depicted in black. The main flat channel runs along the orange dotted line. The axis of rotation is in the middle of the cell at the white spot.

Disorder scattering in classical flat channel transport of particles between twisted magnetic square patterns We measure the trajectories of macroscopic magnetic particles pulled against gravity between twisted alternating magnetic square patterns in a superposed homogeneous magnetic field normal to both patterns [9]. The two patterns are built from a set of magnetic cubes having a distribution of magnetization. The magnetic potential between the patterns is a sum of three contributions: two being periodic on two lattices with different magnitude and orientation, and the third random contribution arising from the distribution of magnetization of the cubes. As one varies the twist angle between the two patterns each time the twist angle coincides with a magic twist angle one of the two periodic lattices becomes a sublattices of the other lattice. Simulations of particles moving through patterns with a precise cube magnetization produce pronounced mobility peaks near magic twist angles that are associated with flat channels. Weak random fluctuations of the cube magnetization in the experiment and the simulations cause enhanced random disorder of the potential and reduce the mobility by scattering particles into the interior of the twisted Wigner Seitz cells. The mobility undergoes an Anderson transition from magic to generic behavior as the magnetization disorder increases beyond half of a percent of the cube magnetization.

4 Published Project Results

4.1 Publications with scientific quality assurance

-
- [1] M. Mirzaee-Kakhki, A. Ernst, D. de las Heras, M. Urbaniak, F. Stobiecki, A. Tomita, R. Huhnstock, I. Koch, A. Ehresmann, D. Holzinger and T. M. Fischer, *Gauge invariant and gauge dependent aspects of topological walking colloidal bipeds*, *Soft Matter* **17**, 1663 (2021), doi:10.1039/D0SM01670E.
- [2] T. Lachner, D. de las Heras and T. M. Fischer, *Braiding with magnetic octupoles*, *Phys. Rev. Res.* **3**, 013043 (2021), doi:10.1103/PhysRevResearch.3.013043.
- [3] S. Barthmann and T. M. Fischer, *The geometric phase and the dry friction of sleeping tops on inclined planes*, *J. Phys. Commun.* **5**(8), 085003 (2021), doi:10.1088/2399-6528/ac1874.
- [4] A. Ernst, A. M. E. B. Rossi and T. M. Fischer, *Adiabatic and irreversible classical discrete time crystals*, *SciPost Phys.* **13**, 091 (2022), doi:10.21468/SciPostPhys.13.4.091.
- [5] M. Mirzaee-Kakhki, A. Ernst, D. de las Heras, M. Urbaniak, F. Stobiecki, J. Gördes, M. Reginka, A. Ehresmann and T. M. Fischer, *Simultaneous polydirectional transport of colloidal bipeds*, *Nat. Commun.* (1), 4670 (2020), doi:10.1038/s41467-020-18467-9.
- [6] N. C. X. Stuhlmüller, F. Farrokhzad, P. Kuświk, F. Stobiecki, M. Urbaniak, S. Akhundzada, A. Ehresmann, T. M. Fischer and D. de las Heras, *Simultaneous and independent topological control of identical microparticles in non-periodic energy landscapes*, *Nat. Commun.* (1), 7517 (2023), doi:10.1038/s41467-023-43390-0.
- [7] N. C. X. Stuhlmüller, T. M. Fischer and D. de las Heras, *Enhanced colloidal transport in twisted magnetic patterns*, *Commun. Phys.* **5**, 48 (2022), doi:10.1038/s42005-022-00824-3.
- [8] N. C. X. Stuhlmüller, T. M. Fischer and D. de las Heras, *Colloidal transport in twisted lattices of optical tweezers*, *Phys. Rev. E* **106**, 034601 (2022), doi:10.1103/PhysRevE.106.034601.
- [9] A. M. E. B. Rossi, A. Ernst, M. Dörfler and T. M. Fischer, *Disorder scattering in classical flat channel transport of particles between twisted magnetic square patterns*, *Commun. Phys.* **7**, 24 (2024), doi:10.1038/s42005-023-01512-6.

4.2 Other publications and published results

Some of the results have been highlighted in press releases of the University of Bayreuth.

Gesteuerte Dynamik von Kolloidstäbchen: Bayreuther Physiker entwickeln Grundlagen für Mini-Laboratorien auf Chips, Universität Bayreuth, Pressemitteilung Nr. 124/2020 vom 16.09.2020 (English version)

Auf magnetischen Lab-on-the-chips: Polyglotte Transportbefehle für kolloidale Zweibeiner, Wiley Analytical Science (2021) link to publication

Präzise Steuerung von Kolloiden durch Magnetismus möglich, Universität Bayreuth, Pressemitteilung Nr. 168/2023 vom 6.12.2023 (English version)