Mathematisches Forschungsinstitut Oberwolfach

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Mathematical Aspects of General Relativity

February 9th – February 15th, 2003

The meeting was organised by Gerhard Huisken (Albert Einstein Institute Potsdam), Jim Isenberg (Eugene) and Alan Rendall (Albert Einstein Institute Potsdam). It addressed recent progress in several areas of Mathematical Relativity, in particular new developments in cosmological models, in energy inequalities, solutions of the constraint equations and solutions of Einstein's equations with various forms of matter.

Of particular importance was the influence of new techniques in nonlinear partial differential equations and differential geometry on the understanding of models in general relativity. PDE techniques prove to be of importance for example in the investigation of stability properties of solutions and their asymptotic behaviour, both for large distances and near singularities. The introduction of new mathematical techniques has made it possible to attack a number of important problems from a new point of view.

The participants composed a good mix of researchers from mathematics and theoretical physics and ensured a close interaction between the two fields. The 25 lectures were given in the morning and late afternoon, a number of collaborations were continued and some new ones initiated amongst the 44 participants. Below are the abstracts of the lectures given during the meeting. All participants enjoyed the wonderful atmosphere provided by the institute and its staff in a white winter landscape.

Abstracts

Gauge fixed evolution for the Einstein equations in low regularity

Lars Anderson (University of Miami)

The Einstein vacuum equations $R_{\alpha\beta} = 0$ in 3+1 dimensions are studied in the gauge given by imposing the constant mean curvature time gauge tr$k = t - t_0$, together with the spatial coordinate gauge given by requiring that the identity map Id : $({}^3M, g) \to ({}^3M, \hat{g})$ is harmonic, for a fixed smooth background metric $\hat{g}$. The resulting elliptic–hyperbolic system is shown to be well–posed in $H^{2+\gamma}$. The proof uses a bootstrap argument, based on a $L^p_t L^\infty_x$ Strichartz estimate, valid for vacuum spacetimes with the given regularity. The proof of the Strichartz estimate is reduced, using Littlewood–Paley decomposition and interpolation estimates, to a $L^2 - L^\infty$ decay estimate, which in turn is proved using a Morawetz type energy estimate. The crucial step in proving the estimate is the study of null cones in a rescaled spacetime. In particular, the quadratic nature of the null structure equations plays a central role in the proof.

Dynamical and Isolated Horizons

Abhay Ashtekar (Pennsylvania State University)

The talk summarized of some the recent results on Dynamical and Isolated Horizons and suggested a number of problems at the interface of geometry and analysis whose solutions will shed considerable light on what John Wheeler called the issue of the 'Final State'. The basic expectation, based on the black hole uniqueness theorems, is that, at late stages of black hole formation, the near horizon geometry should approach the geometry near the Kerr horizon. The key question is: Under which assumptions and in what precise sense does this expectation on the 'Final State' hold? In numerical evolution of such space-times, black holes are represented by apparent horizons on Cauchy slices. The world tube of these apparent horizons constitutes a Dynamical Horizon which are space-like. An isolated horizon is an asymptote to a Dynamical Horizon and is null. A great deal is known about geometry and implications of Einstein's equations on both, including an intrinsic characterization of the Kerr isolated horizon. Therefore, it is possible to pose a series of (mostly elliptic) problems whose solutions will characterize all Dynamical Horizons and their asymptotes that can arise in numerical simulations, and answer questions concerning the 'Final State'.

Energy and monotone quantities

Robert Bartnik (University of Canberra)

This talk reviewed various positivity proofs concerning energy, and stressed the importance of monotone quantities. The second variation formula for area of a hypersurface is closely related to both Geroch's monotonicity (used by Huisken-Ilmanen in their proof of the Penrose Conjecture) and also to a parabolic equation used to construct quasi-spherical and similar solutions to the Hamiltonian constraint (scalar curvature). These ideas have been elegantly combined by Shi-Lam to show that the Brown-York mass of a bounded convex region with non-negative scalar curvature, is non-negative and vanishes only for flat domains.

Blowup in the Yang-Mills-Equations

Piotr Bizon (Jagiellonian University, Krakow)
(joint work with Z. Tabor, Yu. Ovchinnikov, and M. Sigal)

I discuss the formation of singularities for the spherically symmetric Yang-Mills equations in $d+1$ dimensional Minkowski spacetime for $d=4$ (the critical dimension) and $d=5$ (the lowest supercritical dimension). Using combined numerical and analytical methods I argue that in both cases solutions starting with large initial data blow up in finite time and the asymptotic profile of blowup is universal. In $d=5$ this profile is given by the stable self-similar solution while in $d=4$ it is given by the instanton. In the latter case the rate of shrinking of the instanton is derived by solving the modulation equation for the scale factor.

$U(1)$ symmetric Einsteinian spacetimes, the unpolarized case

Yvonne Choquet-Bruhat (Université Paris)

The Einstein equations for vacuum spacetimes with spacelike $U(1)$ isometry group are equivalent to an Einstein - wave map system on a 2+1 dimensional manifold $\Sigma \times R$, together with an ordinary differential system for the evolution of the conformal structure of $\Sigma \times \{t\}$ when the surface Σ, supposed to be compact, has genus greater than 1. I prove the existence of future timelike and null such spacetimes, by using gauge conditions and elliptic estimates to determine the 2+1 metric, up to the conformal structure of $\Sigma \times \{t\}$, together with differential equations in Teichmüller space which govern this conformal structure. Corrected energies estimates lead to a decay of the energies of the wave map and its derivatives. The future global existence is obtained by a bootstrap argument. The theorem is an extension of results obtained in collaboration with V. Moncrief in the "polarized" case, where the wave map reduces to a scalar field.

On the dynamics of Gowdy space times

Piotr T. Chrusciel (Université de Tours)
(joint work with Myeongju Chae)

We study the behaviour near the singularity $t=0$ of Gowdy metrics. A solution is said to satisfy a power law blow-up if the theta derivatives of the associated map into hyperbolic space do not blow up faster than $|t|^{\epsilon-1}$, for some positive constant ϵ, when approaching the singularity $t=0$. No solutions of the Cauchy problem are known which do not satisfy a power low decay. We give an explicit self-similar solution which does not satisfy the power law decay, but it does not fit into a Cauchy problem framework. Every solution with a power law decay has a continuous asymptotic velocity function v, and is strongly censored, with curvature blowing up uniformly, except perhaps at points θ at which $v(\theta)=1$.

Consider the set of initial data for solutions satisfying a power law decay and for which $v<1$. We show that this set is open in the set of all initial data; for those solutions v is smooth except perhaps at points at which it crosses zero; all such solutions have "asymptotically velocity term dominated" (AVTD) behaviour.

In recent further work we have shown that the set of AVTD solutions satisfying a certain uniformity condition is open in the set of all solutions, without the $v<1$ restriction, and that for every solution there exists an open dense set $\Omega \subset S^1$ such that the solution displays AVTD behaviour near $\{0\} \times \Omega$.

On the Asymptotics for the Einstein Constraint Equations

Justin Corvino (Brown University, Providence)
(joint work with R. Schoen)

We study the asymptotics of solutions to the vacuum constraint equations. Given asymptotically flat initial data on M^3 for the vacuum Einstein field equation, and given a bounded domain in M, we construct solutions of the vacuum constraint equations which agree with the original data inside the given domain, and are identical to that of a suitable Kerr slice (or identical to a member of some other admissible family of solutions) outside a large ball in a given end. The data for which this construction works is shown to be dense in an appropriate topology on the space of asymptotically flat solutions of the vacuum constraints. In particular, such data evolves to produce a spacetime with particularly nice behaviour at null infinity. This construction generalizes work in [1], where the time-symmetric case was studied.

[1] Corvino, J.: Scalar Curvature Deformation and a Gluing Construction for the Einstein Constraint Equations. Comm. Math. Phys. 214, 137-189 (2000).

[2] Corvino, J., Schoen, R.M.: On the Asymptotics of the Vacuum Einstein Constraint Equations. Preprint.

The internal structure of charged black holes and the problem of uniqueness in general relativity

Mihalis Dafermos (Massachusetts Institute of Technology)

It is a well known fact that the maximal domain of development of Reissner-Nordstrom data has a smooth future boundary. This indicates that gravitational collapse may lead to a loss of predictability for observers entering the black hole. To examine the generality of this phenomenon, a characteristic initial value problem for the Einstein-Maxwell- Scalar Field equations is studied, with data on the event horizon satisfying a power-law decay. Such data are conjectured to arise generically from the collapse of compactly supported scalar fields. For such data, the spacetime that develops is completely understood. In particular, the boundary of the maximal domain of development is proved to be a null singularity along which the Hawking mass blows up yet the metric can be continuously extended beyond. The implications of these results for the strong cosmic censorship conjecture are discussed.

Initial data for black hole collisions

Sergio Dain (Albert Einstein Institute Potsdam)

I describe the construction of initial data for the Einstein vacuum equations that can represent a collision of two black holes. I stress in the main physical ideas.

PDE methods for spacelike hypersurfaces

Klaus Ecker (Freie Universität Berlin)

Maximal (mean curvature zero) hypersurfaces were used in Schoen and Yau's proof of the positive mass theorem and in the reduction of the Penrose inequality problem to a problem about asymptotically flat 3-manifolds with non-negative scalar curvature, which was recently solved by Huisken-Ilmanen and by Bray.

In this talk, we survey existence and classification results for spacelike hypersurfaces, in particular noncompact ones, in Lorentzian manifolds. We review the geodesic completeness estimate of Cheng-Yau which can be interpreted as a local gradient estimate for solutions of an elliptic PDE. Finally, we present mean curvature flow methods, that is parabolic PDE techniques, for the construction of such hypersurfaces.

Newton's theory of spacetime and gravity as a limit of Einstein's GR

Jürgen Ehlers (Albert Einstein Institute Potsdam)

If the basic laws underlying GR are slightly rearranged and reformulated in terms of rescaled metric variables, they remain meaningful if the positive parameter $\lambda \equiv c^{-2}$ is replaced by 0. The resulting equations reproduce Newton's theory including its spacetime structure, if a condition of asymptotic flatness at spatial infinity is imposed. One can, therefore, define a limit relation for sequences of GR solutions to have Newtonian solutions as limits; Many examples illustrate this. This formalism has been used to obtain GR results from Newtonian ones and to set up Newtonian approximations to GR.

[1] Jürgen Ehlers, The Newtonian limit of General Relativity in "Classical Mechanics and Relativity, relationship and consistency", G. Ferrarese (ed., Napoli 1991).

[2] Jürgen Ehlers, Examples of Newtonian limits of relativistic spacetimes, CQG Nr. 14, A119 - A126, 1997.

The inverse mean curvature flow in cosmological spacetimes. Transition from big crunch to big bang

Claus Gerhardt (Universität Heidelberg)

Let N be a cosmological spacetime of dimension $(n+1)$, $N = (a, b) \times \mathcal{S}_0$, that is asymptotically Robertson-Walker and has a big crunch singularity. We consider the inverse mean curvature flow

$$\dot{x} = -H^{-1}\nu$$

with initial hypersurface M_0, where ν is the past directed normal, and $H_{|M_0} > 0$. If N satisfies a so-called *strong volume decay condition*, then the inverse mean curvature flow exists for all time and provides a smooth foliation of the future of M_0.

Let $M(t)$ be the flow hypersurfaces with mean curvature H. Then, we can prove that the rescaled $M(t)$ converge in $C^\infty(\mathcal{S}_0)$ to a homothetic image of $\mathcal{S}_0$.

Moreover, we can define a new universe $\tilde{N}$ by reflection at the big crunch which is a mirror image of the old universe such that the old and the new stress energy tensors are identical, and all equations that are valid in N also hold in the mirror image. In $\tilde{N}$ the singularity is now a *big bang* singularity.

If we rescale the inverse mean curvature flow by introducing a new flow parameter

$$s = -e^{-\gamma t},$$

with a suitable $\gamma > 0$, then, the rescaled flow can be canonically extended to $\tilde{N}$ as a flow which is of class C^∞ in x and of class C^3 in s. Thus, there exists a natural C^2-diffeomorphism defined on

$$(-a, a) \times \mathcal{S}_0, \quad a > 0,$$

which is of class C^∞ outside the singularity.

Self-similarity in the massless Einstein-Vlasov system

Jose M. Martin-Garcia (University of Southampton)
(joint work with C. Gundlach)

The work reported in this talk is motivated by recent numerical investigations searching for critical phenomena in the gravitational collapse of Vlasov (collisionless Boltzmann) matter coupled to General Relativity. They have found no indication of existence of a self-similar intermediate attractor in phase-space, and some, but not concluding, indications of the existence of a static intermediate attractor, key ingredients for having type I or type II critical phenomena, respectively.

We investigate this issue by numerically constructing static solutions (rigorously established by Rein and Rendall) for massive and massless particles, and self-similar solutions (not known before) for massless particles. For zero mass particles we find that the invariance of the spacetime under redistributions of energy-momentum among the particles prevents the existence of isolated intermediate attractors, so that the usual critical phenomenology cannot be realized in the massless case. This result cannot be directly applied to those numerical investigations because they assume massive particles, but serves as an example of a noncritical system.

Null asymptotic behaviour of the Riemann tensor in a class of vacuum spacetimes

Francesco Nicolo (Universita di Roma)
(joint work with S. Klainerman)

In this work by Sergiu Klainerman and myself we show that, assuming that the metric tensor decays as $O(r^{3+\epsilon})$ with its derivatives up to fourth order, on the initial spacelike hypersurface Σ_0, the various components of the Riemann tensor of the spacetime $(\mathcal{M}, g)$, solution of the Einstein vacuum equations with these initial data, have an asymptotic decay along the null outgoing directions consistent with the predictions of the conformal compactification approach and in particular of the "peeling theorem". It is also proved how much of the peeling result is lost when the decay for the metric tensor on Σ_0 is only $O(r^{3-\gamma})$ with $\gamma \in (\frac{3}{2}, 0]$. The result with $\gamma = 0$ is in partial agreement with the discussion on polyhomogeneous expansion.

Stability of Newtonian Galaxies and Stars

Gerhard Rein (Universität Wien)

We consider self-gravitating matter distributions in a Newtonian framework. The matter is modelled either as a collisionless gas or as a perfect, compressible fluid. The former case—the Vlasov-Poisson system—describes a galaxy, the latter case—the Euler-Poisson system— describes a simple star.

Starting with the Vlasov-Poisson system we consider an energy-Casimir functional acting on density functions on phase space, and we show that minimizers of this functional are non-linearly stable steady states. To prove the existence of minimizers we construct a reduced version of the energy-Casimir functional acting on density functions on space in such a way that there is a one-to-one correspondence between the minimizers of the original functional and those of the reduced one. Minimizers of the latter functional are shown to exists by a concentration-compactness technique.

As a bonus minimizers of the reduced functional turn out to be non-linearly stable steady states (static solutions) of the Euler-Poisson system.

On the asymptotics of Gowdy

Hans Ringström (Albert Einstein Institute Potsdam)

The talk concerned the Gowdy spacetimes under the assumption that the spatial hypersurfaces are three tori. The relevant equations are wave map equations with the hyperbolic space as a target. Asymptotic expansions for the solutions have been proposed to describe the behaviour near the singularity. In the talk we presented different types of conditions yielding such asymptotic expansions.

Selfgravitating elastic bodies in Einstein's theory

Bernd Schmidt (Albert Einstein Institute Potsdam)
(joint work with R. Beig)

Presently we only have existence of static asymptotically flat solutions of Einstein's field equations with sources for fluids, which are necessarily spherically symmetric, and Vlasov matter with axial symmetry. There should, undoubtedly, exist static spacetimes with elastic bodies as sources.

We answered this question in Newton's theory in [1] by proving the existence of small, selfgravitating elastic bodies.

The case we are really interested in is Einstein's Theory. The approach we want to use is to perturb from a Newtonian solution to a solution of Einstein's equations.

In 1981 J. Ehlers found a way to write Einstein's field equations with fluid sources containing $\lambda = \frac{1}{c^2}$ as a parameter, such that the PDEs had a well defined limit system for $\lambda \to 0$, which is equivalent to the Newtonian system. In general this limit is "singular" in the sense that the hyperbolic relativistic equations have a limit which is still hyperbolic in the matter variables but elliptic in the variable describing the gravitational field.

In the time independent case, however, the full equations as well as the limiting equations are elliptic. This offers the possibility to perturb non linearly away from a Newtonian solution to a relativistic one.

This approach has actually been used by U.Heilig [2] who gave the first — and up to now only — existence theorem for a stationary, rigidly rotating fluid with small angular velocity.

He perturbed non linearly away from a Newtonian, non rotating solution. ($\omega = 0, \lambda = 0$) The equations are formulated in harmonic coordinates. Then we have to solve:

(1) the "reduced field equation " for the geometry g and the deformation ϕ

$$F(g, \phi, \lambda) = 0$$

(2) the matter equation

$$T(g, \phi, \lambda) = 0$$

(3) the harmonicity condition

$$H(g, \lambda) = 0$$

(1) and (2) are elliptic for a "reasonable "description of elastic matter. Heilig has shown that solutions of (1),(2) satisfying the correct boundary conditions satisfy (3) for sufficiently small λ.

For $\lambda = 0$ we know the existence of solutions from [1]

$$F(g^N, \phi^N, 0) = 0 \ , \quad T(g^N, \phi^N, 0) = 0 \ , \quad H(g^N, \phi^N, 0) = 0$$

As a first step we obtain solutions $(\delta g, \delta\phi)$ of the equations linearized on (g^N, ϕ^N). The linearized equations have "source terms" determine by the Newtonian solution and λ and can be solved uniquely for sufficiently small λ. Next we plan to investigate whether one can obtain solutions of the full equations for small λ via the implicit function theorem.

[1] R. Beig, B.Schmidt, Proc. Roy. Soc. 459, Nr. 2029 (2003).

[2] U.Heilig, Commun. Math. Phys. 166, 475 (1995).

Estimates for Scalar Fields on a Naked Singularity Background

A. Shadi Tahvildar-Zadeh (Rutgers University, Piscataway)

The Reissner-Nordström solution is the static spherically symmetric solution of the Einstein-Maxwell equations $g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \frac{1}{\alpha^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ where $\alpha^2 = 1 - \frac{2m}{r} + \frac{e^2}{r^2}$. In the case $|e| > m$ the singularity at $r = 0$ is naked. We study the Cauchy problem for a spherically symmetric scalar field ψ on this background $\partial_\mu g^{\mu\nu}\partial_\nu\psi = 0$ and obtain spacetime $L^p L^q$ estimates (Strichartz estimates) for ψ in terms of its initial data. This is motivated in part by the study of the problem of stability of the R-N solution as well as by the desire to understand the issues regarding well-posedness in the absence of global hyperbolicity or cosmic censorship. Using the Regge-Wheeler tortoise coordinate r_*, the equation satisfied by ψ can be written as a wave equation on flat spacetime, with a central potential $V(r_*)$. The potential is singular at the origin $r_* = 0$ and behaves like c/r_*^2 both for r_* small and large. We note that the perturbation equations derived by Zerilli and Moncrief for studying the linearized stability of the Reissner-Nordström solution take the form of two wave equations with potentials that are of this same form. In the case of the wave equation with a potential that is *exactly* inverse-square $U(x) = \frac{a}{|x|^2}$ the desired (generalized) Strichartz estimates have been obtained by Burq, Planchon, Stalker and Tahvildar-Zadeh. We show that this result can be generalized to central potentials that only behave like inverse-square, showing that under an additional hypothesis on the potential, namely $\sup_{r>0} r^2(rU(r))_r < \frac{1}{4}$, the Strichartz estimates continue to hold. We then proceed to show that the particular potential V that comes up in the scalar field problem above satisfies the additional hypothesis that we had to make, as long as $|e| \geq 2m$.

Cosmology, Black Holes, and Shock Waves Beyond the Hubble Length

JOEL SMOLLER (UNIVERSITY OF MICHIGAN)
(joint work with Blake Temple)

In this paper, we put forth in a rigorous mathematical setting, a new Cosmological Model in which the expanding Friedmann-Robertson-Walker (FRW) universe emerges from an event more similar to a classical explosion—there is a shock wave at the leading edge of the expansion—than the standard scenario of the Big Bang. We believe that general relativity pretty much forces such a solution on you as soon as you try to relax the assumption in the standard model that the expansion of the galaxies is of infinite extent at each fixed time. (You could say that in our model, the Copernican Principle is replaced by the principle in physics, that nothing is infinite.) Most importantly, in these new models, the explosion is large enough to account for the enormous scale on which the galaxies and the cosmic background radiation appear uniform.

There are a number of remarkable twists that arise in these new GR blast waves. First of all, the shock wave lies beyond one Hubble length from the FRW center, this threshold being the boundary across which the bounded mass lies inside its own Schwarzschild radius—that is, $2M/r > 1$ beyond one Hubble length—and thus the shock wave solution evolves inside a Black Hole. The nature and evolution of the "total mass" inside the Black Hole is unexpected and interesting.

Another interesting consequence is that the entropy condition chooses the explosion over the implosion, (time irreversibility), and also implies that the shock eventually weakens until it emerges from the Black Hole, (through the White Hole event horizon), as a zero pressure Oppenheimer-Snyder solution. Asymptotically, for large time, the explosion settles down to something like a giant supernova of finite mass and extent, but on an enormous scale—a localized mass expanding into an asymptotically flat Schwarzschild spacetime, everywhere outside the Black Hole.

But he biggest surprise to us is that unlike shock matching outside the Black Hole, the equation of state, $p = 1/3\rho$—the equation of state at the earliest stage of Big Bang physics—is mysteriously distinguished at the instant of the Big Bang. For this equation of state alone, the shock wave emerges from the Big Bang at a finite nonzero speed, the speed of light. (The shock wave then decelerates to a sub-luminous wave at all times after the Big Bang.) These solutions describe, in exact formulas, the global dynamics of strong gravitational field solutions of the Einstein equations, and the setting, inside the Black Hole, is pretty much unexplored territory for analysis.

Wave maps with symmetry

MICHAEL STRUWE (EIDGENÖSSISCHE TECHNISCHE HOCHSCHULE ZÜRICH)

Consider wave maps $U : \mathbf{R}^{2+1}_{+} \to N$ with either radial or co-rotational symmetry, blowing up at $t = 0$, $x = 0$. We show that for suitable $t_i \searrow 0$, $R_i \searrow 0$ $(i \to \infty)$ the rescaled sequence

$$u_i(x) = u(t_i, R_i, x)_{(i\to\infty)} \to u_\infty \quad \text{in} \quad H^1_{\text{loc}}(\mathbf{R}^2_0, N)\,,$$

where $u_\infty : \mathbf{R}^2 \to N$ is harmonic with finite energy, non-constant, and symmetric.

The non-existence of such maps with radial symmetry then shows that the Cauchy problem for radially symmetric wave maps is globally well-posed, and similarly the Cauchy problem for co-rotational wave maps to non-compact target surface of revolution.

Perturbations of Spatially Locally Homogeneous Spacetimes

Masayuki Tanimoto (Yale University)

The aim of this talk is to presented the recent results on the linear perturbation analysis and thereby on the asymptotic stability, of some vacuum solutions that are spatially closed and locally homogeneous but anisotropic. The main focus is on the Bianchi III type (Thurston's $H^2 \times R$). One of the basic results about the evolutions of the perturbations of this type is that they decouple in four independent parts for each appropriate mode. Defining appropriate gauge-invariant variables, the wave equations (equivalent to the linearized Einstein equation) for those variables were explicitly presented for each one of the four kinds. Furthermore, based on the 'zero-mode normalization' scheme, it was concluded that the solution is unstable against the linear perturbations (especially against the ones called 'even' kind). Perturbations for the Bianchi II type were also commented, where in particular the differences between classes B (to which Bianchi III belongs) and A (to which Bianchi II belongs) were stressed.

The past attractor in inhomogeneous cosmology

Claes Uggla (University of Karlstad) and John Wainwright (University of Waterloo)

We present a general framework for analyzing inhomogeneous cosmological dynamics. It employs Hubble-normalized scale-invariant variables which are defined within the orthonormal frame formalism, and leads to a formulation of Einstein's field equations as an autonomous system of evolution equations and constraints. This framework incorporates spatially homogeneous dynamics in a natural way as a special case, thereby placing earlier work on spatially homogeneous cosmology in a broader context, and allows us to draw on experience gained in that field using dynamical systems methods. One of our goals is to provide a precise formulation of the approach to the initial spacelike singularity in cosmological models, described heuristically by Belinskiĭ, Khalatnikov and Lifshitz. We show that the evolution equations admit an invariant set, which we call the *silent boundary*, on which the dynamics reduces to that of spatially homogeneous models. We then use our knowledge of spatially homogeneous dynamics to construct an invariant subset of the silent boundary, which we conjecture forms the local past attractor of the evolution equations. We anticipate that this new formulation will provide the basis for proving rigorous theorems concerning the asymptotic behaviour of inhomogeneous cosmological models.

T^2 symmetry – area of the orbits

Marsha Weaver (University of Alberta)

An open problem in General Relativity is global existence in the sense of characterizing the maximal globally hyperbolic development for general classes of initial data. Consider maximal globally hyperbolic solutions of the vacuum Einstein equations with Cauchy surface topologically T^3 and a spatially acting two dimensional isometry group the orbits of which are topologically T^2. Let the area of the T^2 be R. Suppose $\nabla R \neq 0$. In 1997 Berger, Chruściel, Isenberg and Moncrief showed that on the maximal globally hyperbolic development R serves globally as a time coordinate and takes on all values in (R_0, ∞) for some $R_0 \geq 0$. The new result, with Isenberg, is that if the spacetime is non-flat then R takes on all values in $(0, \infty)$.

Static space time metrics with mean geodesic flow: A variational approach

Gershon Wolansky (Israel Institute of Technology)

The talk examined the stability of the Einstein-Vlasov system with spherical symmetry using variational methods.

The AdS/CFT Correspondence and the Uniqueness of the AdS Soliton

Eric Woolgar (University of Alberta)

By an application of the Null Splitting Theorem of G.J. Galloway [math.DG/9909158], G.J. Galloway, S. Surya, and I have been able to take steps towards the proof of the Horowitz-Myers conjecture. Specifically, under suitable asymptotic conditions (smooth toroidal scri, negative mass, convexity of a certain sort), we find that the AdS soliton is the unique Einstein metric of negative mass admitting a hypersurface-orthogonal Killing field that is timelike near infinity [hep-th/0108170, 0204081, 0212079]. The proof proceeds by noting that the universal covering spacetime of the AdS soliton admits a null line, an inextendible (in fact, complete) achronal null geodesic, and indeed so will any spacetime with the same boundary at infinity and negative mass, given our asymptotic conditions. Now the null splitting theorem asserts that spacetimes admitting null lines split geometrically, simplifying the field equations. It is then straightforward to analyse the remaining field equations and show that they admit an essentially unique solution. However, the solution does depend on the choice of the kernel of the mapping, induced by boundary inclusion, that takes the fundamental group of the conformal boundary onto that of the spacetime. If one invokes the AdS/CFT correspondence, D.N. Page has shown that this non-uniqueness leads to a remarkable prediction seemingly quite unrelated to the Einstein equations, which is that Conformal Field Theory on the 3-torus has zero-temperature phase transitions as the conformal structure is varied [hep-th/0205001].

Edited by Jan Metzger

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