Original Paper

Contribution to the size effect on the strength of flat glass

Kurt Blank[1]), Hugo Grüters and Klaus Hackl
Lehrstuhl und Institut für Technische Mechanik, Rheinisch-Westfälische Technische Hochschule, Aachen (FRG)

Dedicated to Prof. Dr. Franz Gebhardt on the occasion of his 60th birthday

The American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) has published a diagram for the required thicknesses of rectangular glass plates with simply supported edges under uniform load. This diagram is based on experimental fracture tests performed with samples of large glass plates homogeneously loaded. The average fracture load of a sample was divided by the safety coefficient 2.5 and the load so defined was regarded as permissible and safe. In the present paper this diagram is analyzed and the maximum principal tensile stresses are calculated for two selected load levels. For large plates (16 m^2) of annealed glass the maximum permissible tensile stress was 20 N/mm^2, for small plates (0.54 m^2) 37 N/mm^2. The difference between the two values can be explained quantitatively through the "size effect" on the strength of glass; however, the stress distribution being nonuniform these calculations must not be performed for the real surface areas of the plates, but for "effective surface areas" which can be defined through Weibull statistics. For tempered glass, too, the permissible maximum tensile stress depends on the size of the plate, but, as the permanent compressive surface stresses of the tempered glass plates tested are unknown, corresponding analysis of the results represented in the ASHRAE diagram [1] was not possible. The maximum principal tensile stress which was regarded as safe is approximately 55 N/mm^2 for the largest plates of tempered glass used in practice.

Zur Frage des Flächeneinflusses auf die Festigkeit von Flachglas

Die American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) veröffentlichte ein Diagramm zur Dickenbemessung von rechteckigen Glasplatten unter homogener Flächenlast bei vierseitiger freier Randauflage, das auf experimenteller Basis, d. h. auf Grund von Biegebruchversuchen an großen Glasplatten, erstellt wurde. Der Mittelwert der Bruchlasten einer Stichprobe wurde durch den Sicherheitskoeffizienten 2,5 dividiert, und die so bestimmte Flächenlast wurde als zulässig angenommen und für das Diagramm verwendet. In der vorliegenden Arbeit werden aus diesem Diagramm für zwei ausgewählte Belastungen die zugelassenen maximalen Hauptzugspannungen bestimmt. Für große Platten (16 m^2) aus gekühltem Floatglas werden danach als maximale Hauptzugspannungen 20 N/mm^2, für kleine Platten (0,54 m^2) 37 N/mm^2 zugelassen. Der Unterschied läßt sich quantitativ mit dem sogenannten „Flächenfaktor der Glasfestigkeit" erklären; da die Spannungsverteilung jedoch inhomogen ist, darf dabei nicht mit den realen Scheibenflächen, es muß vielmehr mit „effektiven Scheibenflächen" gerechnet werden, die mit Hilfe der Weibull-Statistik definiert werden können. Bei vorgespanntem Glas ergibt sich ebenfalls eine Abhängigkeit der zugelassenen maximalen Hauptzugspannung von der Plattengröße, die aber mangels Kenntnis des Vorspannungsgrades nicht entsprechend analysiert werden konnte. Die zugelassene maximale Hauptzugspannung beträgt hier bei den größten praktisch vorkommenden Scheiben etwa 55 N/mm^2.

1. Introduction

The national standards of the European countries for the determination of the thicknesses of glass plates under load vary to a considerable extent, even if some degree of agreement has been reached in the last two decades. For instance, the required thickness of a glass pane sized (3×2) m^2 under a wind load of 960 N/m^2 is 8 mm in Austria and Switzerland, 9 mm in France and 12 mm in Belgium [2].

Consequently, to check the applicability of results from plate bending theories [3] to practical cases, the comparison with standardized specifications cannot give conclusive evidence. Experimental fracture tests on big samples of large rectangular glass panes have to be performed. Some such experiments were carried out recently by Brünner et al. [4 and 5]. Alternatively, relevant experiments reported in literature have to be analyzed.

In [1] a diagram is shown giving the required thicknesses of rectangular glass panes with simply supported edges under uniform pressure as a function of the size of the panes. This diagram (see figure 1) is based on experimental tests. It was established by loading samples of glass panes of equal dimensions to fracture (unfortunately the sample sizes and the statistical scattering of the results are not mentioned); then the average of the fracture loads was divided by the safety coefficient 2.5 and the load so defined was taken as permissible and used for the diagram.

For comparison, two results of similar measurements by Brünner et al. [4 and 5], with 6 mm thick glass plates, are also shown in figure 1. The two points are based on samples consisting of ten specimens each; the mean values of the fracture loads

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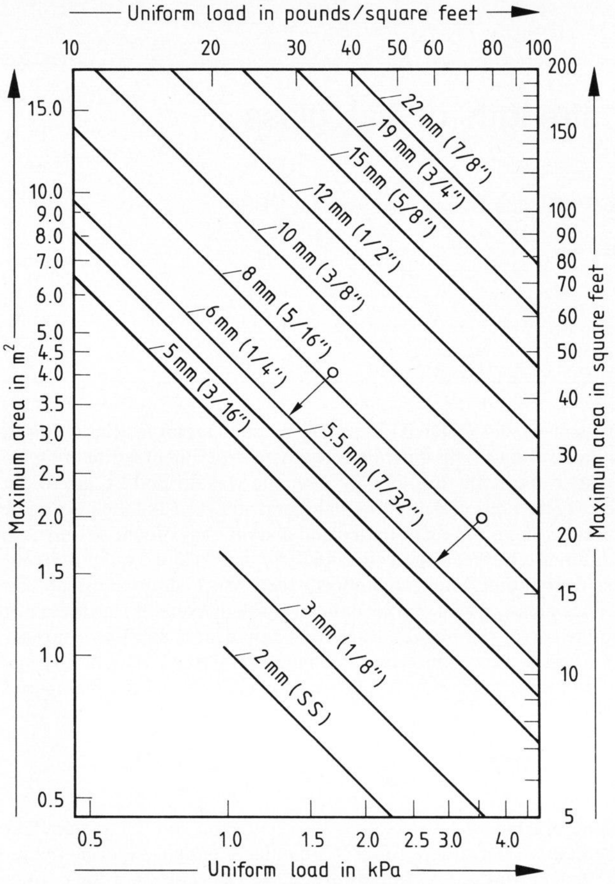
[1]) Haselsteig 13, D-5100 Aachen (FRG).



Multiplying factors for glasses other than annealed

type of glass[2])	multiplying factor
tempered	4.0
heat strengthened	2.0
double glazed	1.5
rough plate	1.0
laminated	0.6
wired	0.5

[2]) Sandblasting will significantly weaken glass, and can reduce its strength by 60 % or more. Sandblasted glass should not be used for exterior glazing.

Figure 1. Permissible maximum size of rectangular float glass plates as a function of uniform load; edges simply supported. Parameter: plate thickness. Multiplying factors for glass other than annealed float glass. Taken from [1]. O: values for a glass thickness of 6 mm [4 and 5].

Table 1. Thicknesses t of glass plates (area $A = 1\ m^2$) necessary for $\sigma_{max} = 20\ N/mm^2$, calculated from the nonlinear theory of shells [3]

edge-length ratio λ	$2a$ in m	t in mm uniform load $p = 1.3\ kN/m^2$	uniform load $p = 3.1\ kN/m^2$
1	1	3.70	6.30
1.5	0.816	3.75	6.69
2	0.707	3.96	6.50
2.5	0.632	4.04	6.45
3	0.577	3.87	6.06
5	0.447	3.13	4.83

were $4.3\ kN/m^2$ for the $(2 \times 2)\ m^2$ and $8.5\ kN/m^2$ for the $(1 \times 2)\ m^2$ samples. The scattering factors were 1.34 and 1.22, respectively, corresponding to standard deviations of approximately 1.1 and $1.6\ kN/m^2$. Like in [1] the mean values of the fracture loads were divided by the safety coefficient 2.5.

The diagram (figure 1) will be analyzed in this paper. The maximum tensile stresses regarded as permissible will be determined by means of a nonlinear plate-bending theory, or, more exactly, by means of a nonlinear theory of shells [3], and their dependence on the size of the plates will be explained through the "size effect" on the strength of glass as predicted by Weibull statistics [6 and 7].

2. Annealed glass

2.1. Permissible maximum tensile stress

The diagram (figure 1) for annealed float glass shows the connection between uniform load, size and thickness of the plate. It is not differentiated as regards the ratios of the edge lengths $\lambda = b/a$, but its use is allowed up to $\lambda = 5$. In the analysis, that value of the ratio λ must be assumed which leads – at given surface area and load – to the highest value of the maximum tensile stress. Within the range of the linear plate-bending theory this holds for $\lambda = 1.5$, e.g. [8]. According to the nonlinear plate-bending theory (nonlinear theory of shells [3]) this statement is no longer generally valid, but in this theory $\lambda = 1.5$ also represents a relatively unfavorable case (table 1). Therefore, the following considerations are based on the ratio of the edge lengths $\lambda = 1.5$.

From figure 1 the maximum stresses occurring, which are also the maximum permissible principal tensile stresses at the surfaces of the plates, were calculated (table 2). Small inconsistencies in table 2 result from the limited accuracy in drawing and reading the diagrams used, and possibly also from the statistical scattering of the test results from which figure 1 has been derived. It is obvious, however, that the maximum tensile stresses resulting from the linear plate-bending theory (table 2, col. 4) are physically wrong, their values being too great, whereas the nonlinear theory of shells [3] leads to reliable results (table 2, col. 9). The differences between the two values of the maximum tensile stresses increase with increasing maximum deformation relative to plate thickness (table 2, col. 6).

Table 2 also makes it clear that a single fixed value of the permissible tensile stress (e.g. $20\ N/mm^2$) would in most cases lead to over-adequate glass thicknesses (table 2, col. 1 with col. 10). This holds especially for relatively small plates. According to table 2, col. 9, maximum tensile stresses up to about $37\ N/mm^2$ are permissible for small plates, whereas only about $20\ N/mm^2$ for the largest ones.

Table 2. Evaluation of the diagram (figure 1) for annealed glass. Assumption: ratio of the edge lengths $\lambda = 1.5$. Col. 9 and 14 to 17: influence of the size of the plates on the permissible maximum tensile stress, σ_{max}, calculated from [3]. Col. 10: plate thickness necessary, if σ_{max} = const. = 20 N/mm²

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
t in mm	A_{max} in m²	$2a$ in m	σ_{max} [3]) in N/mm²	p^*	$\frac{w_{max}}{t}$	w_{max} in mm	σ^*_{max}	σ_{max} [4]) in N/mm²	t_{20} in mm	A_{eff} in m² $\bar{b} = 5$	$\bar{b} = 10$	$\bar{b} = 20$	$\left[\frac{A_{eff,1}}{A_{eff,2}}\right]^{1/\bar{b}}$ $\bar{b} = 5$	$\bar{b} = 10$	$\bar{b} = 20$	$\frac{\sigma_{max,2}}{\sigma_{max,1}}$
uniform load p = 1.3 kN/m²																
2	0.71	0.69	74.9	16.4	6.00	12.0	15.2	35.8	3.2	$4.23 \cdot 10^{-2}$	$6.24 \cdot 10^{-3}$	$4.02 \cdot 10^{-3}$	0.442	0.631	0.871	0.553
3	1.32	0.94	61.9	11.2	4.83	14.5	11.2	31.8	4.3	$9.46 \cdot 10^{-2}$	$1.15 \cdot 10^{-2}$	$7.05 \cdot 10^{-4}$	0.519	0.670	0.799	0.622
5	2.40	1.26	40.5	4.68	2.87	14.4	5.52	24.3	5.8	0.255	$4.01 \cdot 10^{-2}$	$1.42 \cdot 10^{-3}$	0.633	0.759	0.827	0.815
5.5	3.00	1.41	41.9	5.01	2.97	16.3	5.81	24.8	6.5	0.303	$4.80 \cdot 10^{-2}$	$1.57 \cdot 10^{-3}$	0.655	0.773	0.831	0.798
6	3.70	1.57	43.4	5.44	3.12	19.7	6.28	25.7	7.2	0.341	$4.91 \cdot 10^{-2}$	$1.55 \cdot 10^{-3}$	0.671	0.775	0.831	0.770
8	5.05	1.83	33.3	3.18	2.20	17.5	4.00	21.4	8.4	0.734	0.154	$1.00 \cdot 10^{-2}$	0.782	0.869	0.912	0.925
10	7.25	2.20	30.6	2.72	2.00	20.0	3.60	20.8	10.1	1.002	0.233	$1.62 \cdot 10^{-2}$	0.832	0.906	0.934	0.952
12	11.0	2.71	32.2	3.02	2.15	25.8	3.88	21.3	12.5	1.516	0.293	$2.00 \cdot 10^{-2}$	0.904	0.926	0.944	0.930
15	15.5	3.21	29.1	2.43	1.85	27.8	3.24	19.8	14.8	2.512	0.628	$6.35 \cdot 10^{-2}$	1	1	1	1
uniform load p = 3.1 kN/m²																
3	0.54	0.60	60.4	4.43	2.75	8.3	5.27	36.9	4.9	$5.77 \cdot 10^{-2}$	$8.99 \cdot 10^{-3}$	$3.28 \cdot 10^{-4}$	0.429	0.585	0.678	0.610
5	1.02	0.83	41.1	2.05	1.70	8.5	2.87	29.2	6.8	0.167	$4.43 \cdot 10^{-2}$	$4.88 \cdot 10^{-3}$	0.530	0.686	0.776	0.771
5.5	1.29	0.93	42.9	2.23	1.77	9.7	3.08	30.2	7.6	0.203	$4.40 \cdot 10^{-2}$	$4.74 \cdot 10^{-3}$	0.552	0.686	0.775	0.745
6	1.50	1.00	42.0	2.14	1.72	10.3	2.94	29.6	8.2	0.268	$7.27 \cdot 10^{-2}$	$7.98 \cdot 10^{-3}$	0.583	0.721	0.795	0.760
8	2.16	1.20	34.0	1.40	1.32	10.6	2.05	25.5	9.8	0.687	0.374	0.312	0.704	0.850	0.955	0.822
10	3.15	1.40	31.7	1.06	1.12	11.2	1.74	24.9	11.5	1.053	0.702	0.569	0.767	0.905	0.984	0.904
12	4.60	1.75	32.2	1.25	1.25	15.0	1.95	25.7	14.4	1.407	0.631	0.461	0.813	0.895	0.974	0.876
15	6.52	2.09	29.2	1.03	1.07	16.1	1.68	24.2	17.1	2.278	1.512	1.335	0.895	0.977	1.027	0.930
19	8.70	2.41	24.3	0.714	0.85	16.2	1.33	23.1	19.8	3.460	1.900	0.931	0.973	0.999	1.009	0.974
22	11.1	2.72	23.1	0.647	0.77	16.9	1.23	22.5	22.3	3.972	1.911	0.786	1	1	1	1

[3]) Resulting from linear plate-bending theory; [4]) resulting from nonlinear theory of shells.

2.2. Dependence of the permissible maximum tensile stress on the size of the plate

Table 2, col. 9, shows that (neglecting some insignificant irregularities) the permissible maximum principal tensile stress, σ_{max}, decreases systematically with increasing size of the plate or with increasing plate thickness.

As, in reality, the strength of glass is not significantly dependent on its thickness, this correlation must be due to the dependence of its fracture probability on the size of the area under tensile stress, i.e. to the so-called "size effect" on glass strength. In the following it will be checked whether or not these differences of the permissible maximum principal tensile stress can be explained through this assumption.

In [9] the effective principal tensile stresses, σ_{eff}, which determine the fracture probability of a uniformly loaded plate of surface area A have been calculated according to Weibull statistics. Since the surface area A is nonuniformly stressed, σ_{eff} represents a weighted mean value of the major principal tensile stresses; it is defined by equation (1):

$$\sigma_{eff} = \left[\frac{1}{A} \int_A \sigma_p (x, y)^{\bar{b}} \, dx \, dy\right]^{1/\bar{b}} \tag{1}$$

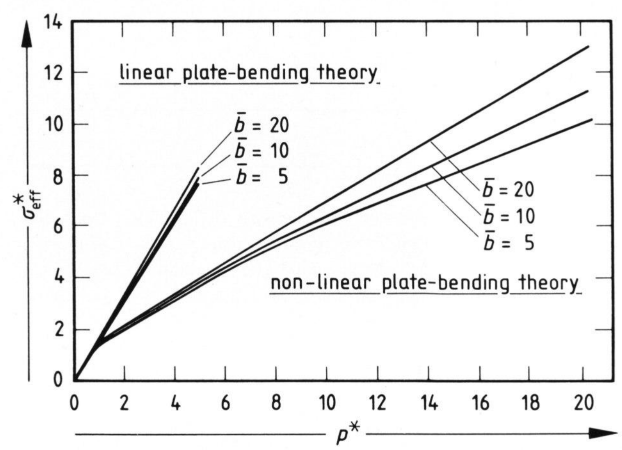
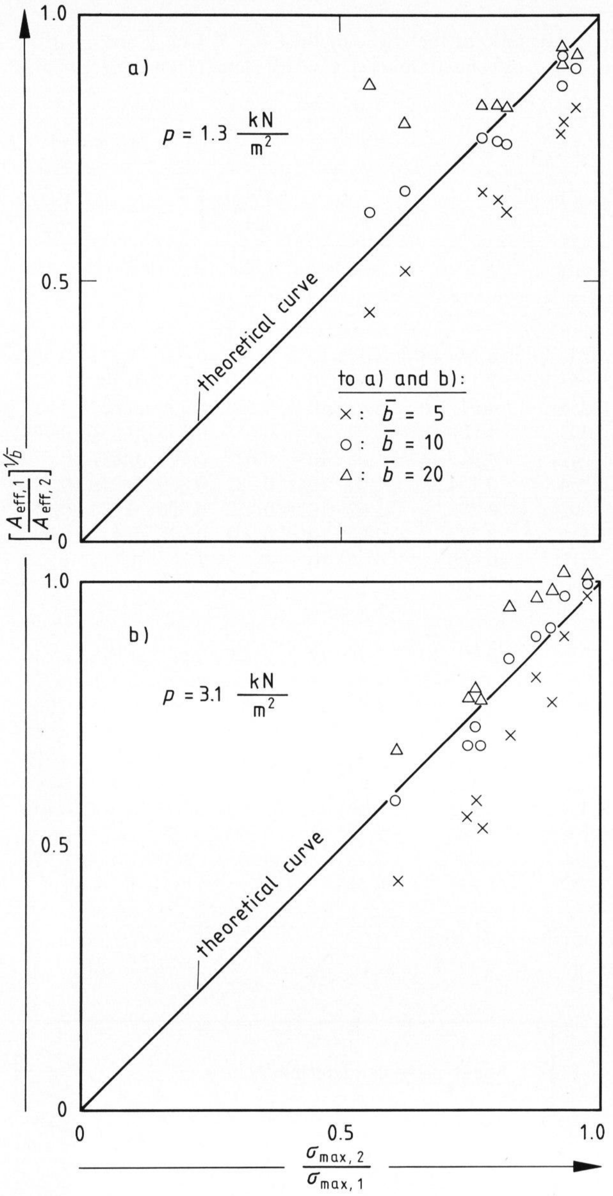


Figure 2. Connection of the effective tensile stress, σ^*_{eff}, with the uniform load, p^*, (normalized dimensionless quantities) for rectangular plates with simply supported edges (edge-length ratio $\lambda = 1.5$).

where $\bar{b}$ is the exponent of the Weibull statistical distribution function [6 and 7].

If normalized quantities are introduced, equation (2) follows:

$$\sigma^*_{eff} = \left[\int_0^1 \int_0^1 \sigma^*_p (\xi, \eta)^{\bar{b}} \, d\xi \, d\eta\right]^{1/\bar{b}}. \tag{2}$$



Figures 3a and b. Influence of the size of annealed float glass plates on the permissible maximum tensile stress. Comparison between experimental results and the prediction by Weibull statistics for two different uniform loads, a), $p = 1.3\ \mathrm{kN/m^2}$, b) $p = 3.1\ \mathrm{kN/m^2}$.

Unfortunately, the calculations of σ^*_{eff} in [9] were carried out only for $\lambda = 1, 2, 3$; therefore, the curves $\sigma^*_{\mathrm{eff}}(p^*)$ for $\lambda = 1.5$ (figure 2) were determined by means of nonlinear interpolation. The following considerations are based on this diagram.

As regards the probability of fracture, the integral

$$\int_A \sigma_{\mathrm{p}}(x, y)^{\bar{b}}\, \mathrm{d}x\, \mathrm{d}y$$

can be interpreted in two different ways:

a) An effective tensile stress, σ_{eff}, is defined which stresses the whole area, A, of the plate uniformly.

b) An effective area, A_{eff}, is defined which is uniformly stressed by the maximum principal tensile stress, $\sigma_{\max}$.

The following equation is obtained:

$$\int_A \sigma_{\mathrm{p}}(x, y)^{\bar{b}}\, \mathrm{d}x\, \mathrm{d}y = A\, \sigma_{\mathrm{eff}}^{\bar{b}} = A_{\mathrm{eff}}\, \sigma_{\max}^{\bar{b}}\,. \tag{3}$$

For constant fracture probability the two different principal tensile stresses, σ_{p}, acting uniformly on two different surface areas, A_1 and A_2, are related to one another according to equation (4), derived from Weibull statistics:

$$\frac{\sigma_{\mathrm{p}}(A_2)}{\sigma_{\mathrm{p}}(A_1)} = \left(\frac{A_1}{A_2}\right)^{1/\bar{b}} \tag{4}$$

As the maximum principal tensile stresses are known (table 2, col. 9), the second interpretation can be used and the connection between A_{eff} and $\sigma_{\max}$ can be determined.

Fracture tests according to the German standard DIN 52 292 [10] with float glass, the surface of which was in its natural state, i.e. not intentionally altered or damaged, resulted in Weibull exponents $\bar{b}$ ranging from 8.8 to 11.6 [11]. Therefore, the calculations of the influence of the "size effect" on the permissible tensile stresses were performed for the three values $\bar{b} = 5, 10, 20$. The results are listed in table 2, col. 11 to 16, and plotted in figures 3a and b. If the Weibull exponent $\bar{b} = 10$ is assumed, the results agree very well with the straight lines in figures 3a and b, predicted by Weibull statistics.

As a consequence, the decrease of the permissible maximum principal tensile stress with increasing surface area of the plate (table 2, col. 9) can be explained with sufficient accuracy through the "size effect" on glass strength, if the value $\bar{b} \approx 10$ is assumed for the Weibull exponent. As mentioned above this value is in good agreement with standardized laboratory fracture tests. It follows that the mathematical expression (equation (4)) for the dependence of the permissible maximum principal tensile stress on the size of the plate can be used for ratios $A_{\mathrm{eff},1}/A_{\mathrm{eff},2}$ up to about 200.

Laboratory tests of glass strength have been performed with various surface areas (between 2.54 and 2500 cm^2) under uniform tensile stress. These tests, too, have led to the conclusion that the Weibull expression for the size factor corresponds with experimental results within the statistical confidence intervals of different samples [11]. The agreement is even better if such tests are carried out with artificially and homogeneously damaged (e.g. abraded) samples. A full agreement is not to be expected because the statistical distribution of a theoretically infinite number of strength values can hardly be represented by one Weibull function covering the whole range from zero to infinity.

Table 3. Evaluation of figure 1 for tempered glass plates. Uniform load: $p = 3.1\ kN/m^2$. Edge-length ratio: $\lambda = 1.5$

1	2	3	4	5	6	7	8	9
t in mm	A_{max} in m^2	$2a$ in m	σ_{max} 5) in N/mm^2	p^*	$\frac{w_{max}}{t}$	w_{max} in mm	σ^*_{max}	σ_{max} 7) in N/mm^2
3	2.16	1.20	242	70.9				
5	4.08	1.66	164	33.6	8.0 6)	40 6)	26.9 6)	68.3 6)
5.5	5.16	1.86	172	36.2	8.1 6)	45 6)	28.3 6)	69.3 6)
6	6.00	2.00	168	34.2	8.0 6)	48 6)	27.1 6)	68.3 6)
8	8.64	2.40	136	22.4	6.95	55.5	19.5	60.7
10	12.6	2.80	127	17.0	6.10	61.0	15.6	55.7
12	18.4	3.50	129	20.0	6.62	79.5	17.7	58.3
15	26.1	4.18	117	16.7	6.07	91.0	15.4	55.5
19	34.8	4.82	97.1	11.5	4.90	93.2	11.5	49.8
22	44.5	5.44	92.6	10.3	4.60	101	10.5	47.5

5) Resulting from linear plate-bending theory; 6) extrapolated values; 7) permissible maximum tensile stress calculated from [3].

3. Tempered glass

The multiplying factors in figure 1 pertain to the maximum permissible size of glass panes; the permissible sizes of panes of annealed glass determined by the diagram have to be multiplied by these factors in cases where other types of glass are to be used. The results for tempered float glass (multiplying factor 4) are listed in table 3. The relevant curves in [3] had partly to be extrapolated; consequently, the maximum deformations and the permissible maximum principal tensile stresses became uncertain at about the sizes of plates important for practical applications.

For tempered glass, too, the permissible maximum principal tensile stress depends on the size of the plates (table 3, col. 9). However, even for the largest plates used in practice σ_{max} is greater than $55\ N/mm^2$.

Unfortunately, it is not possible, from the information available, to analyze the dependence of σ_{max} on the size of the plates as it had been for annealed glass (section 2.2.). To do this, the permanent compressive stress at the surface of the tempered glass plates must be known; this compressive surface stress has to be treated separately from the basic strength of (annealed) glass whenever fracture tests with tempered glass are to be evaluated statistically [12].

4. Conclusions

When the experimental diagram (figure 1) for the determination of the thicknesses of glass plates under uniform load is analyzed, the linear plate-bending theory results in maximum tensile stresses which are certainly too great, whereas the nonlinear plate-bending theory (nonlinear theory of shells) gives reliable values. If the nonlinear theory of shells [3] is used for the calculation of plate thicknesses necessary, the permissible maximum tensile stress should be varied according to the size of the plate. A single value of the permissible maximum tensile stress of e.g. $20\ N/mm^2$ for annealed glass would, for small-sized plates, lead to over-adequate thicknesses.

A dependence of the permissible maximum tensile stress on the size of the plate was found also for tempered glass; however, as the permanent compressive surface stresses were unknown, a detailed analysis of the experimental results was impossible. The assumption of a permissible maximum tensile stress between 55 and $70\ N/mm^2$, according to the size of the plate, is justified according to figure 1.

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5. Nomenclature

5.1. Symbols

A	area of the glass plate
A_{eff}	effective area of the glass plate
A_{max}	maximum permissible area of the glass plate
$2a$	length of the shorter edge of the glass plate
$2b$	length of the longer edge of the glass plate
$\bar{b}$	scattering parameter of the Weibull distribution (Weibull exponent)
E	Young's modulus of elasticity (for float glass $E = 7.0 \cdot 10^{-4}$ in N/mm^2)
p	uniform load (pressure) acting on one side of the glass plate (area A)
t	glass thickness
w	deformation of the glass plate perpendicular to a and b
w_{max}	deformation at the center of the glass plate
x	coordinate in the direction of a
y	coordinate in the direction of b (center of the plate: $x = 0$, $y = 0$)
λ	b/a = ratio of the edge lengths (side ratio)
σ	tensile stress at the surface of the glass plate
σ_{eff}	effective principal tensile stress
σ_p	major principal tensile stress at the surface
σ_{max}	maximum principal tensile stress at the surface

5.2. Normalized, dimensionless quantities

$p^* = p \cdot (a/t)^4 \cdot 1/E$
$\eta = y/b$
$\xi = x/a$
$\sigma^* = \sigma \cdot (a/t)^2 \cdot 1/E$
$\sigma^*_{max} = \sigma_{max} \cdot (a/t)^2 \cdot 1/E$

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